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# Field theory of disordered elastic interfaces at 3-loop order: Critical exponents and scaling functions

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## Abstract

For disordered elastic manifolds in the ground state (equilibrium) we obtain the critical exponents for the roughness and the correction-to-scaling up to 3-loop order, i.e. third order in $\varepsilon = 4 - d$, where $d$ is the internal dimension $d$. We also give the full 2-point function up to order $\varepsilon^2$, i.e. at 2-loop order.


## 1. Introduction

For disordered system the application of the functional renormalization group (FRG) is non-trivial because of the cuspy form of the disorder correlator [1–9]. In [10] we obtained for a 1-component field ($N = 1$) the $\beta$-function to 3-loop order, employing the exact renormalization group and several other techniques. Here we analyze the fixed point: We calculate to 3-loop order the roughness exponent $\zeta$ for random-bond disorder, the universal amplitude for periodic disorder, as well as the RG fixed-point functions and universal correction-to-scaling exponents. We also give the complete functional form of the universal 2-point function up to 2-loop order.

Our results are relevant for a remarkably broad set of problems, from subsequences of random permutations in mathematics [11], random matrices [12,13] to growth models [14–22] and Burgers turbulence in physics [23,24], as well as directed polymers [14,25] and optimization problems such as sequence alignment in biology [26–28]. Furthermore, they are very useful for numerous experimental systems, each with its specific features in a variety of situations. In-

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terfaces in magnets [29,30] experience either short-range disorder (random bond RB), or long range (random field RF). Charge density waves (CDW) [31] or the Bragg glass in superconductors [32–36] are periodic objects pinned by disorder. The contact line of a meniscus on a rough substrate is governed by long-range elasticity [37–41]. All these systems can be parameterized by a $N$-component height or displacement field $u(x)$, where $x$ denotes the $d$-dimensional internal coordinate of the elastic object. An interface in the 3D random-field Ising model has $d=2$, $N=1$, a vortex lattice $d=3$, $N=2$, a contact-line $d=1$ and $N=1$. The so-called directed polymer ($d=1$) subject to a short-range correlated disorder potential has been much studied [42] as it maps onto the Kardar–Parisi–Zhang growth model [14,22,19] for any $N$, and yields an important check for the roughness exponent, defined below, $\zeta_{\mathrm{eq,RB}}(d=1, N=1)=2/3$. Another important field of applications are avalanches, in magnetic systems known as Barkhausen noise. For applications and the necessary theory see e.g. [43–53].

Finally, let us note that the fixed points analyzed here are for equilibrium, a.k.a. "statics". At depinning, both the effective disorder, and the critical exponents change. A notable exception is periodic disorder and its mapping to loop-erased random walks [54], where the disorder force-force correlator $\Delta(u)$ changes by a constant, while all other terms are unchanged, and can be gotten from a simpler scalar field theory, allowing to extend the analysis done here to higher-loop order [54].

## 2. Model and basic definitions

The equilibrium problem is defined by the partition function $\mathcal{Z} := \int \mathcal{D}[u] \exp(-\mathcal{H}[u]/T)$ associated to the Hamiltonian (energy)

$$\mathcal{H}[u] = \int \mathrm{d}^d x \, \frac{1}{2} [\nabla u(x)]^2 + \frac{m^2}{2} [u(x) - w]^2 + V\big(u(x), x\big) \, . \tag{2.1}$$

In order to simplify notations, we will often note

$$\int_x f(x) := \int \mathrm{d}^d x \, f(x) \, , \tag{2.2}$$

and in momentum space

$$\int_q \tilde{f}(q) := \int \frac{\mathrm{d}q^d}{(2\pi)^d} \tilde{f}(q) \, . \tag{2.3}$$

The Hamiltonian (2.1) is the sum of the elastic energy $\int_x \frac{1}{2} [\nabla u(x)]^2$ plus the confining potential $\frac{m^2}{2} \int_x [u(x) - w]^2$ which tends to suppress fluctuations away from the ordered state $u(x) = w$, and a random potential $V(u, x)$ which enhances them. $w$ is, up to a factor of $m^2$, an applied external force, which is useful to measure the renormalized disorder [55,9,56,57,37,52,58], or properly define avalanches [56,57,41,59–62]. The resulting roughness exponent $\zeta$

$$\overline{\langle [u(x) - u(x')]^2 \rangle} \sim |x - x'|^{2\zeta} \tag{2.4}$$

is measured in experiments for systems at equilibrium ($\zeta_{\mathrm{eq}}$) or driven by a force $f$ at zero temperature (depinning, $\zeta_{\mathrm{dep}}$). Here and below $\langle \dots \rangle$ denote thermal averages and $\overline{(\dots)}$ disorder ones. In the zero-temperature limit, the partition function is dominated by the ground state, and we may drop the explicit thermal averages. In some cases, long-range elasticity appears, e.g. for a



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