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Geometric low-energy effective action in a doubled spacetime

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Abstract

The ten-dimensional supergravity theory is a geometric low-energy effective theory and the equations of motion for its fields can be obtained from string theory by computing β functions. With d compact dimensions, an $O(d, d; \mathbb{Z})$ geometric structure can be added to it giving the supergravity theory with T-duality manifest. In this paper, this is constructed through the use of a suitable *star product* whose role is the one to implement the weak constraint on the fields and the gauge parameters in order to have a closed gauge symmetry algebra. The consistency of the action here proposed is based on the orthogonality of the momenta associated with fields in their triple star products in the cubic terms defined for $d \geq 1$. This orthogonality holds also for an arbitrary number of star products of fields for $d = 1$. Finally, we extend our analysis to the double sigma model, non-commutative geometry and open string theory.

1. Introduction

Quantum gravity is expected to unify all theories of the fundamental interactions by combining quantum mechanics and general relativity. String theory is a candidate to provide a

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self-consistent framework for quantum gravity. A crucial role at this aim is played by T-duality and S-duality.

String theory, whose action is defined in the two-dimensional world-sheet space, explores the target-space theory and its low-energy limit via one-loop β-functions [1,2] and α' corrections. One-loop β-functions provide the equations of motion satisfied, in the target space, by the background fields with which the string interacts and, in particular, the one associated with the graviton field, present in the spectrum of closed strings, generates the Einstein gravity equation.

The main goal of double field theory (DFT) [3–6] is to manifestly incorporate T-duality, i.e. the $O(d, d; Z)$ invariance in the target space with d compact dimensions, as a global symmetry of the low-energy field theory deriving from closed strings living in a D-dimensional spacetime which is the product of a Minkowski n-dimensional flat space $\mathcal{M} = R^{n-1,1}$ with a d-dimensional torus T^d $(n + d = D)$. Then, the fields of DFT live in the product of $\mathcal{M}$ with a $2d$-dimensional doubled torus containing both the torus T^d, parametrized by the original compact coordinates x^m, and its dual $\tilde{T}^d$, parametrized by the dual coordinates $\tilde{x}_m$. The field content of DFT involves the metric field g_{ij}, the Kalb–Ramond field B_{ij} $(i, j = 1, \dots D)$ and a dilaton, i.e. the massless bosonic sector of the closed string. Since these fields depend on x^m and $\tilde{x}_m$ simultaneously, DFT is expected to have gauge invariance both under diffeomorphisms on the former and dual diffeomorphisms on the latter, i.e. a gauge invariance under doubled diffeomorphisms.

DFT is still to be fully constructed. In [3], an action for such theory is given only to cubic order in the fluctuations of the above mentioned fields of the massless sector of closed strings around a fixed background. In this framework, the invariance under linearized doubled diffeomorphisms is based on the so-called *weak constraint*, i.e. $\Delta f \equiv 2\partial_m\tilde{\partial}^m f = 0$, that has to be satisfied by fields and gauge parameters that are, therefore, required to live in the kernel of the second-order differential operator Δ. The weak constraint has a stringy interpretation since it arises from the level matching condition of closed string theory. The gauge parameters are the vector fields $\xi^i(x^\mu, x^m, \tilde{x}_m)$ and $\bar{\xi}_i(x^\mu, x^m, \tilde{x}_m)$ generating, respectively, the linearized gauge transformations on the metric tensor and the Kalb–Ramond fields. Subsequently, in [7] a manifestly background independent action has been constructed for the field $\mathcal{E}_{ij} = g_{ij} + B_{ij}$ and for the dilaton d. Further aspects of this action have been studied in [8]. Such formulation results to be $O(D, D)$ invariant and it has been shown equivalent to the generalized metric formulation [9], still $O(D, D)$ invariant, where the invariance under doubled diffeomorphisms is based, this time, on the so-called *strong constraints*, i.e. a generalization of the weak constraint to any product of fields and gauge parameters. In the case of d compact dimensions, the $O(D, D)$ symmetry structure breaks to the $O(d, d; \mathbb{Z})$ symmetry preserving the periodic boundary conditions.

So far, a non-trivial theory that is invariant under doubled diffeomorphisms without using the above mentioned constraints has not been found yet. One could try to formulate a theory in terms of fields automatically projected in the kernel of Δ through a suitably defined projector operator, the *star product*, which takes an arbitrary field or a gauge parameter to that kernel. This operator has to be used also in the gauge transformations in order to ensure that the gauge variations are allowed variations of the fields and shows, in general, a non-associative property that does not give any problems to the invariance of the action that, instead, could be spoiled by the non-closure of the gauge algebra. In other words, it may happen that, due to the non-closure of the gauge algebra, one should not neglect a total derivative term generated by applying a gauge transformation. The total derivative term or boundary term is also related to the global geometry of DFT [10–12]. Furthermore, a closed gauge algebra in DFT is also important to ensure the closure of the supersymmetry algebra [13–15].