Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 931 (2018) 262–282

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Gauge-invariant flow equation

C. Wetterich

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

Received 29 July 2016; accepted 26 April 2018
Available online 30 April 2018
Editor: Stephan Stieberger

Abstract

We propose a closed gauge-invariant functional flow equation for Yang–Mills theories and quantum gravity that only involves one macroscopic gauge field or metric. It is based on a projection on physical and gauge fluctuations. Deriving this equation from a functional integral we employ the freedom in the precise choice of the macroscopic field and the effective average action in order to realize a closed and simple form of the flow equation.

1. Introduction

Functional flow equations permit to interpolate continuously from the microscopic or classical action to the macroscopic or quantum effective action. For Yang Mills theories and quantum gravity local gauge symmetries play a central role. A functional renormalization approach to such theories should keep carefully track of gauge symmetries and resulting restrictions on the general form of the effective action.

The goal is to realize a gauge-invariant effective action $\bar{\Gamma}(\bar{g})$ for a single metric $g_{\mu\nu}$ in gravity, or a single gauge potential $A_\mu = g_\mu$ for electromagnetism, once all fluctuations are taken into account. (Quantum) field equations are then obtained directly as $\partial\bar{\Gamma}/\partial\bar{g} = K$, with K an appropriate conserved source. These field equations are the basis for the "classical" field theories of gravity and electromagnetism, which are well tested by many precision observations. A similar gauge-invariant effective action will be formulated for Yang–Mills theories. Physical correlations

E-mail address: c.wetterich@thphys.uni-heidelberg.de.

https://doi.org/10.1016/j.nuclphysb.2018.04.020
0550-3213/

or Green's functions are obtained by inverting the second functional derivative of $\bar{\Gamma}$ in the space of physical fluctuations.

At the present stage, the formulation of functional flow equations for gauge theories has to deal with the problem that regularization in continuum field theories typically breaks the gauge symmetry, necessitating gauge fixing. Furthermore, quadratic infrared cutoff terms are usually not compatible with the gauge symmetry. Exact flow equations for the effective average action of gauge theories have been formulated in the background field formalism [1–4]. This formalism has been extended to quantum gravity [5]. These flow equations involve, however, two independent fields. The first is the expectation value of the microscopic or fluctuating field g', over which the functional integral is performed,

$$g = \langle g' \rangle, \tag{1}$$

while the second "background field" $\bar{g}$ is used to formulate covariant derivatives for the gauge fixing and infrared cutoff. The effective action is only invariant under simultaneous transformations of g and $\bar{g}$.

Alternatively, one may omit the background field, which amounts to setting $\bar{g} = 0$ in the background field formalism. The effective action is no longer gauge invariant. Rather sophisticated approximation schemes [6,7] are needed in order to cope with the many terms contributing already in low orders of the gauge field. It has been proposed to maintain gauge symmetry by the use of rather complex gauge invariant regularizations [8–11] involving additional fields. Our present approach is more modest. Technically, it shows analogies to background gauge fixing in a particular "physical" gauge. We obtain, however, a gauge invariant effective action depending only on one macroscopic gauge field. This is achieved by employing the macroscopic field for the formulation of the gauge fixing and infrared regulator term. No separate background field is introduced. At the end, we obtain indeed a quantum effective action that is gauge invariant and depends on a single metric or gauge field. This can be used as the basis for general relativity and Maxwells equations, including corrections to these equations generated by quantum fluctuations.

In the usual "background field formalism" $\bar{g}$ is considered as fixed. We propose here to replace the fixed background field by a macroscopic field $\bar{g}(g)$, with a relation to g that is, in principle, computable. The macroscopic field $\bar{g}$ is the argument of the gauge-invariant effective action $\bar{\Gamma}(\bar{g})$ which only depends now on one field. The metric or gauge field in the field equations is identified with $\bar{g}$. Also the flow equations describe the scale dependence of the effective action at fixed $\bar{g}$. Thus $\bar{g}$ is the relevant field for all macroscopic considerations. (We keep here the notation $\bar{g}$ for comparison with the background field formalism – the bar may be dropped at later stages.) The choice of the relation between $\bar{g}$ and $g = \langle g' \rangle$ is such that a closed gauge invariant flow equation can be formulated for $\bar{\Gamma}(\bar{g})$. The precise relation between g and $\bar{g}$ is of secondary importance.

Approximative solutions (truncations) of previous versions of exact flow equations for gauge theories have been successfully used to understand various phenomena. Superconductivity or the abelian Higgs model has been investigated in various dimensions [2,4,12,13]. Increasingly sophisticated truncations in quantum chromodynamics (QCD) provide for an increasingly complete analytical understanding [14–16]. Functional renormalization has addressed the running of the gauge coupling in various dimensions [1,3,17–19]. Applied to thermal equilibrium, with an effective non-perturbative ("confinement") scale increasing with temperature, it has been advocated that non-perturbative strong interaction effects should be visible in the quark gluon plasma even at high temperature [1,20]. (This qualitative finding has been made quantitative by computations of thermodynamic quantities in lattice gauge theories, or by the experimental observation of strong interaction properties in heavy ion collisions at high effective temperature.) Detailed