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Nuclear Physics B 929 (2018) 1–20
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Multi-cut solutions in Chern–Simons matrix models

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Received 20 July 2017; accepted 27 January 2018
Available online 5 February 2018
Editor: Leonardo Rastelli

Abstract

We elaborate the Chern–Simons (CS) matrix models at large N. The saddle point equations of these matrix models have a curious structure which cannot be seen in the ordinary one matrix models. Thanks to this structure, an infinite number of multi-cut solutions exist in the CS matrix models. Particularly we exactly derive the two-cut solutions at finite 't Hooft coupling in the pure CS matrix model. In the ABJM matrix model, we argue that some of multi-cut solutions might be interpreted as a condensation of the D2-brane instantons.

1. Introduction

Matrix models play crucial roles in various topics in theoretical physics. (See, for example, recent reviews [1,2].) Particularly, in string theory, there are several proposals that matrixes may provide the non-perturbative formulation of non-critical strings [3–6] and critical strings [7–9], and, many remarkable results have been obtained in this direction. Besides, since matrixes are also related to the large-N gauge theories, they play special roles in quantum gravity through the gauge/gravity correspondence [10–13].

Recently, the importance of matrix models is significantly increasing in supersymmetric gauge theories and related mathematical physics too. In supersymmetric theories, the dynamical degree of freedom drastically reduce due to the cancellation between the bosons and fermions, and, in

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https://doi.org/10.1016/j.nuclphysb.2018.01.028
0550-3213/

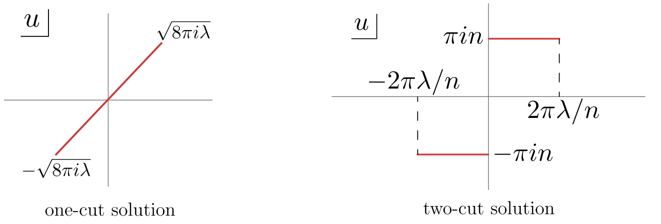

Fig. 1. Solutions of the pure CS matrix model at weak coupling $|\lambda| \ll 1$. The red lines represent the eigenvalue distributions (cuts). n in the two-cut solution is an integer. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

some special cases, the theories are effectively described by zero dimensional matrix models [14–17]. Especially, through the development of the localization technique [18,19], many correspondences between the supersymmetric gauge theories and matrix models have been found. (See reviews [20–22].)

Among these matrix models, we focus on the $U(N)$ Chern–Simons (CS) matrix models in this article. The partition function of the pure CS matrix model is given by [23–25]

$$Z_N^{\mathrm{CS}}(k) = \frac{1}{N!}\int \prod_i \frac{du_i}{2\pi} e^{-\frac{N}{4\pi i\lambda}\sum_i u_i^2} \prod_{i<j}\left[2\sinh\frac{u_i - u_j}{2}\right]^2. \tag{1}$$

Here $\lambda \equiv N/k$ is the 't Hooft coupling and k is an integer representing the CS level. The computation of this integral has been already done and we know the exact value of this partition function as a function of k and N [25,26]. However investigating this model is still valuable. As we will see, this model shows curious properties which have not been seen in any other one matrix models. In addition, the varieties of the pure CS matrix model are being actively studied, and understanding this model may help us in developing insights into these models. For example, the ABJM theory [13] on S^3 is described by a similar matrix integral [25]. Also the three dimensional $\mathcal{N} = 2$ supersymmetric CS matter theory coupled to matters with arbitrary R-charge on S^3 is described by a related matrix model [27,28]. Especially the 't Hooft limit of these models are important in string theory, and we study these CS matrix models under this limit.

When we take the 't Hooft limit ($N \to \infty$, λ fixed), the saddle point approximation may be applicable. The saddle point equation of the pure CS matrix model (1) is given by

$$u_i = \frac{2\pi i\lambda}{N}\sum_{j\neq i}\coth\frac{u_i - u_j}{2}, \qquad (i = 1, \cdots, N). \tag{2}$$

The solution of this equation at finite λ has been found, and it is characterized by a single cut of the eigenvalue distribution [24,29,30]. (See Fig. 1 (left).)

Then a natural question is whether the solution is unique. We find that the answer is no. The saddle point equation (2) has an interesting structure which allows an infinite number of solutions characterized by multi-cut eigenvalue distributions. See Fig. 1 (right) for a two-cut solution. Moreover these multi-cut solutions would ubiquitously exist in the varieties of the CS matrix models too. (Actually such multi-cut solutions were first found in the CS matrix model coupled to adjoint matters through a numerical analysis [31].)

In one matrix models, the multi-cut solutions are usually related to the large-N instantons [32,33]. We argue that indeed some of the multi-cut solutions in the ABJM theory might be interpreted as a condensation of the D2-brane instantons [34]. Besides, some of the multi-cut