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An anomalous propulsion mechanism

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Abstract

We consider a gas of free chiral fermions trapped inside a uniform rotating spherical shell. Once the shell becomes transparent the fermions are emitted along the axis of rotation due to the chiral and mixed anomaly. In return, owing to momentum conservation, the shell is propelled forward. We study the dependence of the magnitude of this effect on the shell parameters in a controlled setting and find that it is sensitive to the formation of an ergosphere around the rotating shell. A brief discussion on a possible relation to pulsar kicks is provided.

1. Introduction

Chiral fermions are anomalous—what seems like a $U(1)$ symmetry of the free action is not a symmetry of the partition function. This peculiar property has had many repercussions on modern-day particle physics ranging from an explanation of the abnormally large decay rate of the neutral pion [1,2] through restrictions on possible extensions of the standard model [3] to duality relations between supersymmetric quantum field theories [4]. Over the last forty odd years much effort has gone into a comprehensive classification of anomalies and an understanding of their effect on S-matrix elements and vacuum correlation functions (see e.g., [5] for a comprehensive review). More recently, it has been established that anomalies play a prominent role in

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the response of the system to vorticity (or a magnetic field) when the system is in or near thermal equilibrium [6–13].

Let us briefly review these latest developments. The (covariant) energy momentum tensor $T^{\mu\nu}$ and chiral current J^μ satisfy the (non-)conservation laws

$$\begin{aligned}\nabla_\mu J^\mu &= \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\left(3c_A F_{\mu\nu}F_{\rho\sigma} + c_m R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\sigma}\right)\\ \nabla_\mu T^{\mu\nu} &= F^\mu{}_\nu J^\nu + \frac{c_m}{2}\nabla_\nu\left(\epsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}\right)\end{aligned} \tag{1}$$

where

$$c_A = -\frac{1}{24\pi^2} \qquad c_m = -\frac{1}{192\pi^2}\,, \tag{2}$$

are the anomaly coefficients for a left handed fermion and indicate the strength of the anomaly, $R^\alpha{}_{\beta\gamma\delta}$ is the Riemann tensor and $F = dA$ is a flavor field strength. Our conventions are the same as those specified in [12], where (1) has been carefully derived. Had $c_A = c_m = 0$, (1) would have reduced to current conservation and energy–momentum conservation up to a Joule heating term.

When the system is near thermodynamic equilibrium one finds that the 10 components of the stress tensor and four components of the chiral current depend on five parameters: a temperature field T, a chemical potential field μ, and a velocity field u^μ normalized such that $u^\mu u_\mu = -1$. The expression relating the components of the stress tensor and current to the five thermodynamic parameters are referred to as constitutive relations. Due to possible field redefinitions of the thermodynamic parameters there is some non physical ambiguity in the constitutive relations which may be fixed by an appropriate definition of thermodynamic fields (see e.g., [14–16]). Choosing a particular definition is referred to as a choice of frame. In what follows we will use a particular definition, referred to as the thermodynamic frame [17] (used also in [18]). We point out that our final results are frame independent.

The constitutive relations for the energy momentum tensor, to first order in derivatives, in the thermodynamic frame, are given by

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P(g^{\mu\nu} + u^\mu u^\nu) + u^\mu q^\nu + u^\nu q^\mu - \eta\sigma^{\mu\nu} \tag{3a}$$

where $P(T,\mu)$ is the pressure, $\epsilon = -P + T\frac{\partial P}{\partial T} + \mu\frac{\partial P}{\partial \mu}$,

$$q^\mu = -2\left(8\pi^2 c_m \mu T^2 + c_A\mu^3\right)\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma \tag{3b}$$

and

$$\sigma^{\mu\nu} = (u^\mu u^\rho + g^{\mu\rho})(u^\nu u^\sigma + g^{\nu\sigma})\left(\nabla_\rho u_\sigma + \nabla_\sigma u_\rho\right) - \frac{1}{3}\left(u^\mu u^\nu + g^{\mu\nu}\right)\nabla_\rho u^\rho\,. \tag{3c}$$

The constitutive relations for the anomalous $U(1)$ current are given by

$$J^\mu = \rho u^\mu + \nu^\mu\,, \tag{3d}$$

where ρ is a charge density and ν^μ is given by

$$\nu^\mu = \sigma(u^\mu u^\rho + g^{\mu\rho})\left(E_\rho - T\nabla_\rho\frac{\mu}{T}\right) - (8\pi^2 c_m T^2 + 3c_A\mu^2)\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma \tag{3e}$$

where $E^\mu = F^{\mu\nu}u_\nu$ is the flavor electric field in the rest frame of a fluid element. We refer the reader to [11] for a detailed derivation of (3). Solving (1) with (3) will provide us with an