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Monodromy relations in higher-loop string amplitudes

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Abstract

New monodromy relations of loop amplitudes are derived in open string theory. We particularly study N-point (planar and non-planar) one-loop amplitudes described by a world-sheet cylinder and derive a set of relations between subamplitudes of different color orderings. Various consistency checks are performed by matching α'-expansions of planar and non-planar amplitudes involving elliptic iterated integrals with the resulting periods giving rise to two sets of multiple elliptic zeta values. The latter refer to the two homology cycles on the once-punctured complex elliptic curve and the monodromy equations provide relations between these two sets of multiple elliptic zeta values. Furthermore, our monodromy relations involve new objects for which we present a tentative interpretation in terms of open string scattering amplitudes in the presence of a non-trivial gauge field flux. Finally, we provide an outlook on how to generalize the new monodromy relations to the non-oriented case and beyond the one-loop level. Comparing a subset of our results with recent findings in the literature we find therein several serious issues related to the structure and significance of monodromy phases and the relevance of missed contributions from contour integrations.

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1. Introduction

The study of perturbative scattering amplitudes in string theory has been a very active field of research in recent years. In particular, a better understanding of the properties, structure and symmetries of string amplitudes has shed light on similar quantities in gauge and gravity theories. In fact, many results for scattering amplitudes in $\mathcal{N}=4$ supersymmetric Yang Mills (SYM) theory were first derived in string theory [1,2]. Studying the geometric properties of the string world-sheet proves to be very fruitful along these directions. Important relations between objects in field theory such as the Bern–Carrasco–Johansson (BCJ) or Kawai–Lewellen–Tye (KLT) relations have been derived from string theory. Indeed, these relations follow from monodromy properties of the tree-level string world-sheet [3–5]. Non-trivial monodromies are related to vertex operator positions on the string world-sheet and branch cuts originating from the universal Koba–Nielsen factor accounting for the plane wave correlator in the string amplitudes. Studying the resulting monodromy contours provides interesting mathematical identities and physical constraints.

More concretely, we start with the color decomposition of the tree-level N-point open string amplitude $\mathfrak{A}_N^{(0)}$ of N adjoint gluons [6]. The latter can be decomposed w.r.t. the color ordered subamplitudes as

$$\mathfrak{A}_N^{(0)} = g^{N-2} \sum_{\sigma\in S_{N-1}} \mathrm{Tr}(T^{a_1} T^{a_{\sigma(2)}} \ldots T^{a_{\sigma(N)}})\, A^{(0)}(1,\sigma(2),\ldots,\sigma(N))\,, \tag{1.1}$$

with the gauge coupling constant g and the Chan–Paton factors T^a being generators of the $U(N_c) \supset SU(N_c)$ color group. The single trace structure is invariant under cyclic transformations. As a consequence the first entry can be fixed and the sum is over all non-cyclic permutations $S_{N-1} = S_N/\mathbf{Z}_N$. In the following we shall consider the color-ordered open string N-point tree-level subamplitude $A^{(0)}(1,2,\ldots,N)$. The latter is described by a world-sheet disk which contains N ordered vertex operator insertions at the boundary of the disk (cf. the left part of Fig. 1). For a given color-ordering all vertices are cyclically ordered, i.e. their relative ordering is kept fixed. In order to obtain relations between different orderings, we consider moving one vertex insertion (e.g. the vertex labeled by 1) along the contour shown in red in the right part of Fig. 1. As indicated, this contour is chosen in a specific way to avoid the branch points arising when the vertex 1 collides with any of the other vertex operator insertions on the boundary. Thus, since the integrand of the subamplitude $A^{(0)}(1,2,\ldots,N)$ is a holomorphic function, which is integrated along a closed contour and encloses no poles,[1] the resulting integral vanishes. This amounts to the following tree-level identity [4,5]

$$\begin{aligned} A^{(0)}(1,2,\ldots,N) + e^{i\pi s_{12}} A^{(0)}(2,1,\ldots,N) + e^{i\pi s_{1,23}}\, A^{(0)}(2,3,1,4,\ldots,N) \\ + \ldots + e^{i\pi s_{1,2\ldots N-1}}\, A^{(0)}(2,3,\ldots,N-1,1,N) = 0\,, \end{aligned} \tag{1.2}$$

where

$$s_{ij} = \alpha'\,(k_i+k_j)^2 = 2\alpha' k_i\cdot k_j\,, \quad s_{i,j\ldots l} = 2\alpha' k_i(k_j+\ldots+k_l)\quad, \quad \sum_{i=1}^N k_i = 0\,, \tag{1.3}$$

[1] Here we do not consider additional closed string bulk insertions. For a generalization of the discussion including closed string insertions, see [7].