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Explicit calculation of multi-fold contour integrals of certain ratios of Euler gamma functions. Part 1

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Abstract

In this paper we proceed to study properties of Mellin–Barnes (MB) transforms of Usyukina–Davydychev (UD) functions. In our previous papers (Allendes et al., 2013 [13], Kniehl et al., 2013 [14]) we showed that multi-fold Mellin–Barnes (MB) transforms of Usyukina–Davydychev (UD) functions may be reduced to two-fold MB transforms and that the higher-order UD functions were obtained in terms of a differential operator by applying it to a slightly modified first UD function. The result is valid in $d = 4$ dimensions and its analog in $d = 4 - 2\varepsilon$ dimensions exits too (Gonzalez and Kondrashuk, 2013 [6]). In Allendes et al. (2013) [13] the chain of recurrent relations for analytically regularized UD functions was obtained implicitly by comparing the left hand side and the right hand side of the diagrammatic relations between the diagrams with different loop orders. In turn, these diagrammatic relations were obtained due to the method of loop reduction for the triangle ladder diagrams proposed in 1983 by Belokurov and Usyukina. Here we reproduce these recurrent relations by calculating explicitly via Barnes lemmas the contour integrals produced by the left hand sides of the diagrammatic relations. In such a way we explicitly calculate a family of multi-fold contour integrals of certain ratios of Euler gamma functions. We make a conjecture that similar results for the contour integrals are valid for a wider family of smooth functions which includes the MB transforms of UD functions.

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1. Introduction

Off-shell triangle-ladder and box-ladder diagrams are the only family of the Feynman diagrams which were calculated at any loop order, for example in $d = 4$ space–time dimensions [1–4] with all indices equal to 1 in the momentum space representation (m.s.r.) and in $d = 4 - 2\varepsilon$ space–time dimensions with indices equal to $1 - \varepsilon$ on the rungs of ladders in the m.s.r. too [5,6]. For the important case of the ladder diagrams with all indices equal to 1 in the m.s.r. in $d = 4 - 2\varepsilon$ space–time dimensions the on-shell result for this family of diagrams is known only at the first three loops in the form of expansion in terms of ε [7,8] up to a certain power of ε. The off-shell result for the whole family of the ladder diagrams is unknown in $d = 4 - 2\varepsilon$ dimension.

The momentum integrals corresponding to the family of the ladder diagrams in $d = 4$ space–time dimensions result in UD functions [2,3]. The order of the UD function is the loop order in the ladder diagram [2,3,9]. The ladder diagrams possess remarkable properties at the diagrammatic level, for example, in Refs. [10,11] it was shown that the UD functions are invariant with respect to Fourier transformations. In Ref. [12,9] it has been shown that such a property of Fourier invariance may be generalized to any three-point Green function via Mellin–Barnes transformation.

MB transforms of the UD functions were investigated in Refs. [13,14]. It has been found under some analytical regularization of Ref. [1] that MB transform of n-order UD function is a linear combination of MB transforms of three UD functions of $(n - 1)$-order. This means any ladder diagram of this family may be reduced via a chain of recurrent relations to the one-loop scalar massless triangle diagram, which may be expressed for any indices and in any dimensions in terms of Appell function F_4 [15,16]. This chain of the recurrent relations for the analytically regularized UD functions in the double-uniform limit when removing this analytical regularization, is represented as a differential operator applied a to a slightly modified first UD function [14]. It has been shown there that if instead of MB transforms of UD functions we write any smooth function of the same arguments the structure of this differential operator will be maintained the same in this double uniform limit. This operator will be applied to the function of the lowest order in this chain of recurrent relations.

However, in the present paper we show that in the particular case when in the integrand of the contour integrals on the left hand sides of the diagrammatic relations the MB transforms of the UD functions stand, this chain of recurrent relations for the MB transforms of UD functions is produced by the contour integration. These contour integrals are calculated explicitly via the first and the second Barnes lemmas. Due to observation done in the previous paragraph, we make a conjecture that similar results for the contour integrals are valid for a wider family of smooth functions written instead of MB transforms of UD functions. In the next papers we describe this family of functions and also describe what kind of changes should be made for the contours of the integrals over complex variables for the case of other smooth functions different from certain ratios of Euler gamma functions. In this paper we focus on the contour integration via Barnes lemmas for the case when the integrand contains MB transforms of UD functions.

The Barnes lemmas were introduced in science about century ago. The first Barnes lemma has been proved in Ref. [17], the second Barnes lemma has been proved in Ref. [18]. They allow

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