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Assessment of plate theories for free-edge effects

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ABSTRACT

This paper deals with a comparative study of several laminated plate theories with respect to their capability to capture the steep transverse stress gradients occurring in vicinity of free edges. The considered laminated plate theories pertain to the family of Equivalent Single Layer (ESL) as well as Layer-Wise (LW) descriptions. Reference is made to the classical displacement-based approach as well as to a partially mixed variational formulation, which allows to introduce independent assumptions for the transverse stresses and the displacements. Finite element solutions are obtained for free-edge effects that arise in several representative laminates subjected to uniaxial tension. An equivalent stress measure is proposed for assessing the three-dimensional (3D) stress fields predicted by the various theories. It is shown that refined LW models can provide quasi-3D results that compare well with full 3D FEM computations, whereas ESL models fail to capture the free-edge effects. Present results indicate that free-edge effects induced by a ±45° interface are most critical for the accuracy of laminated plate models.

1. Introduction

Fiber-reinforced composites are being increasingly used in weight-sensitive industrial applications, in particular aerospace structures. Composite plate and shell structures basically consist of a stack of several plies whose fiber directions are oriented at different angles. Computational and analytical tools for predicting the behavior of these structures all rely on the so-called effective modulus theory (EMT), in which the heterogeneous fiber–matrix system is homogenized at ply level. As a consequence, elasticity solutions as well as approximated models represent the composite structure as a stack of different homogeneous and anisotropic materials, in which adjacent layers are separated by bi-material interfaces.

Over the last 40 years, much effort has been dedicated to the formulation of appropriate plate/shell models that should cope with the peculiar displacement, strain and stress fields produced by the stiffness distribution across the laminate's cross-section. An overwhelming amount of literature has been produced on this topic, as witnessed by the numerous review articles over composite plate/shell models, e.g., [1–7]. Extensions of models originally formulated for homogeneous structures basically lead to a structural homogenization in which single ply's properties are smeared into an equivalent laminate stiffness. These so-called Equivalent Single Layer (ESL) models have a number of unknown parameters that does not depend on the number of layers constituting the stack. The most widely employed plate models, namely Classical Laminated Plate Theory (CLPT) and First-order Shear Deformation Theory (FSDT), belong to this group [8]. However, these models cannot represent the response at bi-material interfaces where, according to EMT, the transverse stresses need to be continuous (interlaminar equilibrium) and the displacement field has consequently a discontinuous slope along the thickness direction. So-called Zig-Zag models can represent up to a certain extent these interfaces still within an ESL approach [9]. Within EMT, the most refined description considers each ply's properties separately; in these Layer-Wise (LW) models, the number of unknown parameters depends on the number of the represented layers. Three-dimensional (3D) elasticity solutions make evidently use of this layer-wise description.

It is nowadays well established that the simplest 2D models, namely CLPT and FSDT, can be effectively used only for a stiffness design that accounts for the laminate's gross response. If a more accurate representation of the stress field is demanded, for instance for a strength design, refined models should be employed. An assessment permitting clear statements concerning the pertinence of a given model with respect to the investigated response appears thus of major interest. As can be seen in the already cited review articles, most of the papers proposing assessments of 2D plate/shell models refer to rather academic benchmark problems that are essentially limited to the global bending, buckling or free-vibration response and for which exact 3D elasticity solutions may be available as solid reference. However, the main discriminating feature for reduced 2D models is their capability to accurately represent transverse/interlaminar stresses [10]. Configurations with known transverse stress risers appear, hence, as natural candidates for assessing plate models.

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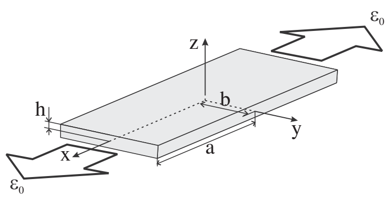

Fig. 1. The Pipes–Pagano problem: composite laminate under uniaxial uniform tensile load.

It is known that relevant transverse stresses arise in presence of in-plane stress gradients and in vicinity of material and/or geometric discontinuities, such as ply-drops, cut-outs and stress-free edges, see, e.g., [11]. Due to its simple implementation within the experimental setup illustrated in Fig. 1, the free-edge effect arising in a composite plate subjected to tensile loads has been extensively studied since the seminal works of Pipes and Pagano [12,13]. Free-edge effects are typical boundary layer effects where a 3D stress concentration is locally confined in a small region in the vicinity of the free edge [14]. While the simple plane stress solution of CLPT holds in the inner region of the plate, no exact elasticity solution is up to now known for the complex 3D stress state at the stress-free edge. Due to the presence of bi-material interfaces, some stress components show even a singular behavior [15–17]. As a consequence, mesh-dependent stress fields are predicted in vicinity of the free edge, which renders questionable the use of these stress components for, e.g., delamination initiation [18]. Dedicated mesh refinement strategies or element formulations have been proposed in the framework of the Finite Element Method (FEM) in order to measure the singularity, see, e.g., [19]. Stress intensity factors can thus be evaluated that may be further used in a delamination onset analysis as in, e.g., [20]. In this context, an attractive alternative to FEM is given by the Boundary Element Method, which appears as an efficient method for the singularity analysis because it permits to substantially reduce the computational effort by confining the dense mesh into the boundary region of concern [21].

Based on the above discussion, the objective of the present paper is to numerically assess a large number of plate models with respect to their capability of representing the stress concentration occurring in the Pipes–Pagano problem illustrated in Fig. 1. The main feature of 2D models is to avoid the cumbersome mesh generation that comes along 3D FEM analysis of general laminate configurations, in which the discretization in the plane (x,y) of the structure depends on that utilized along the laminate's thickness direction z. Most of the assessed models are obtained in the framework of Carrera's Unified Formulation (CUF), which is a compact notation that allows to easily formulate the governing equations for different 2D models [22]. CUF includes ESL and LW models based on either the classical displacement-based approach or the partially mixed approach of Reissner, in which independent assumptions are introduced for the displacement and the transverse stress field [23,24]. It may be interesting to note that the first FEM application of Reissner's partially Mixed Variational Theorem (RMVT) was proposed by Pian and Li and included ESL and LW descriptions for the analysis of stress fields around a hole [25]. Thanks to the systematic procedure of CUF, hierarchic 2D models can be formulated and have been thoroughly assessed in several papers with respect to free-vibration, bending and buckling response of laminated plates and shells, see, e.g., [26–29]. However, an assessment of these models with respect to the free-edge problem, as proposed in the present work, is still missing. In addition to CUF models, the present contribution employs a refinement of the classical sinus model proposed by Touratier [30], which includes transverse normal stress by retaining a quadratic through-the-thickness variation of the transverse displacement as formulated in [31,32].

The outline of this paper is as follows: Section 2 proposes a selective review of several approaches to the Pipes–Pagano problem. CUF models and their FEM implementation are briefly recalled in Section 3 along with the refined sinus model. In Section 4, present numerical results for the Pipes–Pagano problem are first compared with those available in literature. Subsequently, an equivalent stress measure is proposed for the quantitative assessment of the models. The study considers symmetric cross-ply, angle-ply and quasi-isotropic laminates. Furthermore, the paper presents results of 3D FEM computations performed with the commercial software Ansys, against which results from plate models are thoroughly compared. Conclusions are finally summarized in Section 5 along with an outlook towards future investigations.

2. Selected literature review of the Pipes–Pagano problem

An overwhelming amount of literature has been devoted to the classical Pipes–Pagano problem illustrated in Fig. 1. Since an exhaustive review is out of the scope of the present investigation, we refer the interested reader to the more complete surveys of this topic by Mittelstedt and Becker [33,34]. The following selected review is essentially limited to those works whose results are included in the numerical study of the present paper.

In absence of an exact elasticity solution for the free-edge field, approximate solutions have been proposed in semi-analytical closed-form or by means of numerical approximations. The semi-analytical solution approach of Tahani and Nosier [35] solves in closed-form the boundary layer in the direction perpendicular to the stress-free edge (the y direction of Fig. 1), while an approximate solution is found along the through-thickness direction (the z direction of Fig. 1). The latter is defined according to Reddy's layer-wise theory [8] upon discretizing the thickness of each ply in several mathematical layers, in which the field variables are interpolated through linear Lagrange polynomials. Successive refinement can be achieved by increasing the number of mathematical layers in which the plies are divided into (h refinement along the thickness). In this approach, the stress-free conditions at the free edge are thus enforced in an averaged sense by means of weighted integrals. The results of Tahani and Nosier [35] are obtained with 15 subdivisions per ply.

Except the first numerical analysis of the Pipes–Pagano problem provided in [12], which is based on a finite difference scheme, most of the numerical reference solutions have been obtained by means of the Finite Element Method (FEM). Wang and Crossman employed generalized plane strain, three-node triangular elements for discretizing the mid-section $x = 0$ of the laminate [36]. Thanks to the Sky-line storage scheme for the stiffness matrix, Wang and Crossman could use a denser mesh and, hence, provide accurate reference results despite the rather poor computer power available at that time. Subsequent work using the same classical displacement-based FEM focused on the stress singularity at the intersection between the bi-material interface and the free edge [15], and showed that accurate solutions could be found everywhere except in the elements closest to the free edge, i.e., in a region that can be made arbitrarily small upon mesh refinement [37]. The aforementioned works relied on the quasi-3D model (Q3D), which refers to the hypothesis of zero gradients along the axial coordinate x and retaining an axial warping of the (y,z)-planes which depends only on y and z (axis notation according to Fig. 1).

The classical displacement-based method suffers some inherent limitations, in particular the discontinuity of the transverse stress field at the bi-material interface and the approximate satisfaction of the stress-free boundary conditions. So, stress-based equilibrium approaches have been proposed in conjunction with either analytical [38] or numerical FEM-based solutions [39]. Starting