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Renormalization Group: Applications in Statistical Physics

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Abstract

These notes aim to provide a concise pedagogical introduction to some important applications of the renormalization group in statistical physics. After briefly reviewing the scaling approach and Ginzburg–Landau theory for critical phenomena near continuous phase transitions in thermal equilibrium, Wilson's momentum shell renormalization group method is presented, and the critical exponents for the scalar Φ^4 model are determined to first order in a dimensional ϵ expansion about the upper critical dimension $d_c = 4$. Subsequently, the physically equivalent but technically more versatile field-theoretic formulation of the perturbational renormalization group for static critical phenomena is described. It is explained how the emergence of scale invariance connects ultraviolet divergences to infrared singularities, and the renormalization group equation is employed to compute the critical exponents for the $O(n)$-symmetric Landau–Ginzburg–Wilson theory to lowest non-trivial order in the ϵ expansion. The second part of this overview is devoted to field theory representations of non-linear stochastic dynamical systems, and the application of renormalization group tools to critical dynamics. Dynamic critical phenomena in systems near equilibrium are efficiently captured through Langevin stochastic equations of motion, and their mapping onto the Janssen–De Dominicis response functional, as exemplified by the field-theoretic treatment of purely relaxational models with non-conserved (model A) and conserved order parameter (model B). As examples for other universality classes, the Langevin description and scaling exponents for isotropic ferromagnets at the critical point (model J) and for driven diffusive non-equilibrium systems are discussed. Finally, an outlook is presented to scale-invariant phenomena and non-equilibrium phase transitions in interacting particle systems. It is shown how the stochastic master equation associated with chemical reactions or population dynamics models can be mapped onto imaginary-time, non-Hermitian "quantum" mechanics. In the continuum limit, this Doi–Peliti Hamiltonian is in turn represented through a coherent-state path integral action, which allows an efficient and powerful renormalization group analysis of, e.g., diffusion-limited annihilation processes, and of phase transitions from active to inactive, absorbing states.

Keywords:
renormalization group, critical phenomena, critical dynamics, driven diffusive systems,
diffusion-limited chemical reactions, non-equilibrium phase transitions
2008 MSC: 82-01, 82B27, 82B28, 82C26, 82C27, 82C28, 82C31

1. Introduction

Since Ken Wilson's seminal work in the early 1970s [1], based also on the groundbreaking foundations laid by Leo Kadanoff, Ben Widom, Michael Fisher [2], and others in the preceding decade, the renormalization group (RG) has had a profound impact on modern statistical physics. Not only do renormalization group methods provide a powerful tool to analytically describe and quantitatively capture both static and dynamic critical phenomena near continuous phase transitions that are governed by strong interactions, fluctuations, and correlations, they also allow us to address physical prop-

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erties associated with the emerging generic scale invariance in certain entire thermodynamic phases, many non-equilibrium steady states, and in relaxation phenomena towards either equilibrium or non-equilibrium stationary states. In fact, the renormalization group presents us with a conceptual framework and mathematical language that has become ubiquitous in the theoretical description of many complex interacting many-particle systems encountered in nature. One may even argue that the fundamental RG notions of universality and relevance or irrelevance of interactions and perturbations, and the accompanying systematic coarse-graining procedures are of crucial importance for any attempt at capturing natural phenomena in terms of only a few meso- or macroscopic degrees of freedom, and thus also form the essential philosophical basis for any computational modeling, including Monte Carlo simulations.

In these lecture notes, I aim to give a pedagogical introduction and concise overview of first the classic applications of renormalization group methods to equilibrium critical phenomena, and subsequently to the study of critical dynamics, both near and far away from thermal equilibrium. The second half of this article will specifically explain how the stochastic dynamics of interacting many-particle systems, mathematically described either through (coupled) non-linear Langevin or more "microscopic" master equations, can be mapped onto dynamical field theory representations, and then analyzed by means of RG-improved perturbative expansions. In addition, it will be demonstrated how exploiting the general structure of the RG flow equations, fixed point conditions, and prevalent symmetries yields certain exact statements. Other authors contributing to this volume will discuss additional applications of renormalization group tools to a broad variety of physical systems and problems, and also cover more recently developed efficient non-perturbative approaches.

2. Critical Phenomena

We begin with a quick review of Landau's generic mean-field treatment of continuous phase transitions in thermal equilibrium, define the critical exponents that characterize thermodynamic singularities, and then venture to an even more general description of critical phenomena by means of scaling theory. Next we generalize to spatially inhomogeneous configurations, investigate critical infrared singularities in the two-point correlation function, and analyze the Gaussian fluctuations for the ensuing Landau–Ginzburg–Wilson Hamiltonian (scalar Euclidean Φ^4 field theory). This allows us to identify $d_c = 4$ as the upper critical dimension below which fluctuations crucially impact the critical power laws. Finally, we introduce Wilson's momentum shell renormalization group approach, reconsider the Gaussian model, discuss the general emerging structure, and at last perturbatively compute the fluctuation corrections to the critical exponents to first order in the dimensional expansion parameter $\epsilon = d_c - d$. Far more detailed expositions of the contents of this chapter can be found in the excellent textbooks [3]–[8] and in chap. 1 of Ref. [9].

2.1. Continuous phase transitions

Different thermodynamic phases are characterized by certain macroscopic, usually extensive state variables called order parameters; examples are the magnetization in ferromagnetic systems, polarization in ferroelectrics, and the macroscopically occupied ground-state wave function for superfluids and superconductors. We shall henceforth set our order parameter to vanish in the high-temperature disordered phase, and to assume a finite value in the low-temperature ordered phase. Landau's basic construction of a general mean-field description for phase transitions relies on an expansion of the free energy (density) in terms of the order parameter, naturally constrained by the symmetries of the physical system under consideration. For example, consider a scalar order parameter ϕ with discrete inversion or Z_2 symmetry that in the ordered phase may take either of two degenerate values $\phi_\pm = \pm|\phi_0|$. We shall see that the following generic expansion (with real coefficients) indeed describes a *continuous* or second-order phase transition:

$$f(\phi) = \frac{r}{2}\,\phi^2 + \frac{u}{4!}\,\phi^4 + \ldots - h\,\phi\,, \tag{1}$$

if the temperature-dependent parameter r changes sign at T_c. For simplicity, and again in the spirit of a regular Taylor expansion, we let $r = a(T - T_c^0)$, where T_c^0 denotes the *mean-field critical temperature*. Stability requires that $u > 0$ (otherwise more expansion terms need to be added); near the critical point we can simply take u to be a constant. Note that the external field h, thermodynamically conjugate to the order parameter, *explicitly* breaks the assumed Z_2 symmetry $\phi \to -\phi$.

Minimizing the free energy with respect to ϕ then yields the thermodynamic ground state. Thus, from $f'(\phi) = 0$ we immediately infer the *equation of state*

$$h(T,\phi) = r(T)\,\phi + \frac{u}{6}\,\phi^3\,, \tag{2}$$

and the minimization or stability condition reads $0 < f''(\phi) = r + \frac{u}{2}\,\phi^2$. At $T = T_c^0$, (2) reduces to the *critical isotherm* $h(T_c^0,\phi) = \frac{u}{6}\,\phi^3$. For $r > 0$, the *spontaneous*

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