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Wilson-loop formalism for Reggeon exchange at high energy

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Abstract

I will discuss how the non-vacuum, quark-antiquark Reggeon-exchange contribution to meson-meson elastic scattering, at high energy and low tranferred momentum, can be related to the path-integral of a certain Wilson-loop expectation value over the trajectories of the exchanged fermions. Making use of this representation, I will show how a linear Regge trajectory is obtained through gauge/gravity duality and the use of minimal surfaces.

Keywords: Nonperturbative effects, QCD, Gauge-gravity correspondence

1. Introduction

The study of soft high-energy scattering (SHES) in strong interactions ($\sqrt{s} \gg 1$ GeV, $\sqrt{|t|} \lesssim 1$ GeV) dates back to the mid '50s, well before the discovery of QCD. One of the key concepts in the phenomenological description of SHES is that of Regge poles, i.e., singularities in the complex-angular-momentum plane of the t-channel amplitude, corresponding in physical terms to the exchange of families of states between the colliding hadrons. The position of these singularities varies with t along the so-called Regge trajectories $\alpha(t)$, and governs the high-energy behaviour of the scattering amplitudes, i.e., $\mathcal{A}(s \to \infty, t) \sim s^{\alpha(t)}$. The dominant trajectory in the elastic channel is called *Pomeron*, and corresponds to the exchange of states with vacuum quantum numbers, while the subleading non-vacuum trajectories are usually called *Reggeons*. The explanation of these concepts from first principles is an open problem in QCD, involving its nonperturbative (NP), strong-coupling regime.

A NP approach to SHES in the framework of QCD has been proposed some time ago [1]. This approach adopts a partonic description of hadrons over a small time-window at interaction time, over which partons do not split or annihilate and can be treated as in and out states of a scattering process. Starting from the corresponding amplitudes, one then reconstructs the hadronic amplitudes by folding with appropriate wave functions describing the hadrons. In the case of the Pomeron exchange (PE) process, the partons travel approximately on their classical trajectories, as the energy is large, and are practically undisturbed by the diffusion process, as the momentum transfer is small, and so can be treated in an eikonal approximation [1–3]. In the case of Reggeon exchange (RE) the picture involves the exchange of a pair of valence quark and antiquark, and a different treatment is required. An approach based on the path-integral (PI) representation for the fermion propagator [4] and on analytic continuation (AC) to Euclidean space [5–7] has been suggested a few years ago [8], but a complete derivation was lacking until recently [9].

2. Reggeon Exchange and Wilson Loops

We briefly sketch now the derivation of a NP expression for the RE contribution to SHES amplitudes. We focus on the elastic scattering of two mesons $M_{1,2}$, of masses $m_{1,2}$, taken for simplicity with the following flavour content, $M_1 = Q\bar{q}$, $M_2 = q\bar{Q}'$. In the soft high-energy regime, the initial momenta $p_{1,2} = m_{1,2}u_{1,2}$, with u_i purely longitudinal, $u_i^2 = 1$ and $u_1 \cdot u_2 = \cosh\chi$, are practically unchanged by the scattering process, i.e., $p'_i \simeq p_i$, with the transferred momentum $q = p_1 - p'_1 \simeq (0, 0, \vec{q}_\perp)$.

The starting point is adopting a description of the mesons as wave packets of transverse colourless dipoles [2], so reducing the meson-meson S-matrix, S_{fi}, to the dipole-dipole (dd) S-matrix, $S^{(dd)}_{fi}$,

$$S_{fi} = \int d\mu\, S^{(dd)}_{fi}(\mu)\,, \quad d\mu = d\mu'^{*}_1 d\mu'^{*}_2 d\mu_1\, d\mu_2\,, \quad (1)$$

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where μ denotes collectively the various degrees of freedom, and the integration measures $d\mu_i^{(*)}$ are defined as

$$\int d\mu_i^{(*)} \equiv \int d^2 k_{i\perp} \int_0^1 d\zeta_i \sum_{s_i,t_i} \psi^{(*)}_{i\,s_i t_i}(\vec{k}_{i\perp},\zeta_i)\,, \qquad (2)$$

with $\vec{k}_{i\perp}$ and ζ_i the transverse momentum and the longitudinal momentum fraction of the quark in meson i, s_i and t_i the spin indices of the quark and antiquark in meson i, respectively, and ψ_i the wave function for meson i. For later convenience we define also the wave function in coordinate space, $\varphi_{i\,st}(\vec{R}_\perp,\zeta) = \sqrt{2\zeta(1-\zeta)2\pi}\int d^2k_\perp e^{i\vec{k}_\perp\cdot\vec{R}_\perp}\psi_{i\,st}(\vec{k}_\perp,\zeta)$, where $\vec{R}_\perp$ is the transverse size of the dipole. One then performs a LSZ reduction, identifying PE with the parton-elastic process, and RE with the parton-inelastic one (Fig. 1):

$$S_{fi}^{(dd)}(\mu) = \mathcal{P}^{(dd)}(\mu) + \mathcal{R}^{(dd)}(\mu)\,. \qquad (3)$$

The PE contribution has been investigated in a number of papers [1–3, 10–15], and will not be discussed here.

As regards RE, at this point one exploits the representation of the fermion propagators as PIs of Wilson lines running along the trajectories of the partons [4], and the spacetime picture of the process to identify the dominant contributions to the PI in the large s, small t regime. In the initial stage of the process a "wee" (i.e., carrying a vanishingly small fraction of longitudinal momentum) valence quark q in meson 2, and a "wee" valence antiquark $\bar{q}$ in meson 1, enter the interaction region along the classical straight-line trajectories of the mesons, then "bend" their trajectory, and annihilate producing gluons; in the final stage of the process, these gluons produce a "wee" $q\bar{q}$ pair, whose components rejoin the "spectator" partons to form the mesons in the final state.[1] As for the "spectator" partons, which carry a relevant fraction of longitudinal momentum, they travel almost undisturbed along their eikonal trajectories. This suggests that only those paths that coincide with the incoming and outgoing eikonal trajectories of the exchanged partons at early and late times contribute to the path integral.

At this point one has to work out the details, taking care of removing the internal interactions of the two mesons, that do not take part in the scattering process. Defining the RE contribution to elastic meson-meson scattering $\mathcal{A}_\mathcal{R}$, $\mathcal{R} = \int d\mu\,\mathcal{R}^{(dd)}(\mu) = i(2\pi)^4\delta^{(4)}(p_f - p_i)\mathcal{A}_\mathcal{R}$, one obtains the following expression for $\mathcal{A}_\mathcal{R}$,

$$\mathcal{A}_\mathcal{R}(s,t) = \lim_{\zeta_1\to 1,\,\zeta_2\to 0} \int d\tilde{\mu}_{\zeta_1,\zeta_2}\mathcal{A}_\mathcal{R}^{(dd)}(s,t;\tilde{\mu})\,, \qquad (4)$$

[1] Things can go also in the reverse order, with the production of a fermion-antifermion pair preceeding the annihilation.

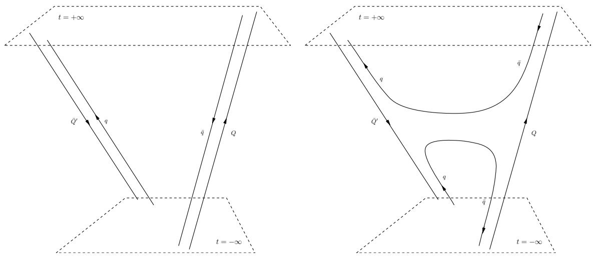

Figure 1: Space-time picture of the Pomeron-exchange (PE) process (left) and of the Reggeon-exchange (RE) process (right).

where the new integration measure $d\tilde{\mu}_{\zeta_1,\zeta_2}$ is

$$\int d\tilde{\mu}_{\zeta_1,\zeta_2} = \int d^2R_{1\perp}\int d^2R_{2\perp}\int d^2R'_{1\perp}\int d^2R'_{2\perp} \times \sum_{t'_1,t_1,s'_2,s_2} \rho_{1\,t'_1t_1}(\vec{R}_{1\perp},\vec{R}'_{1\perp},\zeta_1)\rho_{2\,s'_2s_2}(\vec{R}_{2\perp},\vec{R}'_{2\perp},\zeta_2)\,, \qquad (5)$$

where $\rho_{i\,r'r}(\vec{R}_{i\perp},\vec{R}'_{i\perp},\zeta_i) = \sum_s \varphi^*_{i\,sr'}(\vec{R}'_{i\perp},\zeta_i)\varphi_{i\,sr}(\vec{R}_{i\perp},\zeta_i)$ with $\vec{R}^{(\prime)}_{i\perp}$ the dipole size in the initial (final) state, and $\mathcal{A}_\mathcal{R}^{(dd)}(s,t;\tilde{\mu})$ the RE contribution to dd scattering,

$$\begin{aligned}\mathcal{A}_\mathcal{R}^{(dd)}(s,t;\tilde{\mu}) =& -i2s\frac{(2\pi)^2}{m_1m_2}\frac{1}{N_c}\int d^2b_\perp\, e^{i\vec{q}_\perp\cdot\vec{b}_\perp} \\ &\times \int \mathcal{D}C^{(\wedge)}\int\mathcal{D}C^{(\vee)}\, e^{i4m_qT}e^{-i(m_q-i\epsilon)(L^{(\wedge)}+L^{(\vee)})} \\ &\times \mathrm{S}_\wedge^{t_1s_2}[\dot{C}^{(\wedge)};p_{\bar{q}},p_q]\mathrm{S}_\vee^{s'_2t'_1}[\dot{C}^{(\vee)};p'_q,p'_{\bar{q}}]\mathcal{U}_C[C^{(\wedge)},C^{(\vee)}]\,.\end{aligned} \qquad (6)$$

Here we have denoted $C^{(\wedge),(\vee)} = (L^{(\wedge),(\vee)}, X^{(\wedge),(\vee)})$, $\dot{C}^{(\wedge),(\vee)} = (L^{(\wedge),(\vee)},\dot{X}^{(\wedge),(\vee)})$, with $\int\mathcal{D}C^{(\wedge),(\vee)}$ defined as

$$\int\mathcal{D}C^{(\wedge),(\vee)} = \int_{2T-L_0}^{2T+L_0} dL^{(\wedge),(\vee)}\int_{x_i^{(\wedge),(\vee)}}^{x_f^{(\wedge),(\vee)}}[\mathcal{D}X^{(\wedge),(\vee)}]\,; \qquad (7)$$

$L^{(\wedge),(\vee)}$ is the length of path $X^{(\wedge),(\vee)}$, with endpoints[2]

$$\begin{aligned} x_i^{(\wedge)} &= -u_2T + \tfrac{R_2}{2}\,, & x_f^{(\wedge)} &= -u_1T + b - \tfrac{R_1}{2}\,, \\ x_i^{(\vee)} &= u_1T + b - R'_1 + \tfrac{R_1}{2}\,, & x_f^{(\vee)} &= u_2T + R'_2 - \tfrac{R_2}{2}\,, \end{aligned} \qquad (8)$$

where we have set $b = (0,0,\vec{b}_\perp)$, $R^{(\prime)}_{1,2} = (0,0,\vec{R}^{(\prime)}_{1,2\perp})$. In Eq. (6), the quantities

$$\begin{aligned}\mathrm{S}_\wedge^{t_1s_2}[\dot{C}^{(\wedge)};p_{\bar{q}},p_q] &= \frac{\bar{v}^{t_1}(p_{\bar{q}})\mathcal{S}_{-T,-T+L}[\dot{X}^{(\wedge)}]u^{s_2}(p_q)}{2\sqrt{\tilde{m}_q\tilde{m}_{\bar{q}}}}\,, \\ \mathrm{S}_\vee^{s'_2t'_1}[\dot{C}^{(\vee)};p'_q,p'_{\bar{q}}] &= \frac{\bar{u}^{s'_2}(p'_q)\mathcal{S}_{-T,-T+L'}[\dot{X}^{(\vee)}]v^{t'_1}(p'_{\bar{q}})}{2\sqrt{\tilde{m}'_q\tilde{m}'_{\bar{q}}}}\,,\end{aligned} \qquad (9)$$

[2] Note that they have been corrected with respect to Ref. [9].