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Nuclear Physics B (Proc. Suppl.) 234 (2013) 299–302
NUCLEAR PHYSICS B PROCEEDINGS SUPPLEMENTS
www.elsevier.com/locate/npbps

ChPT calculations of pionic formfactors

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Abstract

An overview on chiral perturbation theory calculations of form factors is presented. The main focus is given on the form factors related to the lightest meson, pion, namely: pion decay constant, pion vector and scalar form factor, radiative pion decay and transition form factor. A pure calculation within the effective theory can be extended using further methods, as resonance chiral theory and leading logarithm calculations.

Keywords: Chiral Lagrangians, 1/N Expansion, radiative decay of π^0

1. Introduction

The formfactors of quantum chromodynamics (QCD) are well defined objects which can be studied both theoretically and experimentally. We will focus on several basic quantities which are connected with π meson and summarize basic status of their theoretical calculations mainly at low energies, i.e. at the domain of chiral perturbation theory (ChPT). The formfactors connected for example with kaons will not be considered here, but one should note that they play also important role in connection with ChPT (e.g. $K_{\ell 4}$).

2. Pion decay constant

The most simplest formfactor, pion decay constant, is defined in QCD via the coupling of axial current and pion as

$$\langle 0|A^a_\mu(x)|\pi^b(p)\rangle = i\delta^{ab}F_\pi p_\mu \mathrm{e}^{-ipx}\,. \qquad (1)$$

As the pion is real, $p^2 = m_\pi^2$, the momentum dependence is trivial and F_π is a constant. This is a reason why it is usually not referred as formfactor in the literature (on recent review see e.g. [1] and references therein). It is a fundamental order parameter of the spontaneous symmetry breaking of $SU(N_f)_L \times SU(N_f)_R$ to $SU(N_f)_V$ of QCD (N_f represents number of light quark flavours, 2 or 3 for real QCD). Its value can be set from the $\pi_{\ell 2}$ decay using Marciano and Sirlin formula for radiative corrections [2]. Updated by virtual photons [3] and V_{ud} value [4] one can obtain [5]

$$F_\pi = 92.215 \pm 0.0625\,\mathrm{MeV}\,. \qquad (2)$$

In pure QCD, F_{π^0} and $F_{\pi^\pm}$ difference is NNLO effect and this was evaluated in [5] and found to be indeed very small. We use this fact in order to set the pion decay constant form π^0 lifetime. Using the NNLO calculation within ChPT of $\pi^0 \to \gamma\gamma$ decay [5] subtracting QED corrections one can arrive to

$$F_{\pi^0} = 93.85 \pm 1.3\,(\mathrm{exp.}) \ \pm 0.6\,(\mathrm{theory})\,\mathrm{MeV}\,. \qquad (3)$$

As an experimental input the PrimEx measurement was used [6]. We can see that the precision obtained here cannot still compete with the precision obtained using charged pion decay. However new experimental activity (e.g. PrimEx2, KLOE-II) can improve the experimental error. On the theory side there are also possible improvements foreseen. One of them, the full calculation of the $\eta \to \gamma\gamma$ decay will be valuable [7], as well as a better estimation of the value of the isospin breaking coefficient $\sim (m_d - m_u)$. What is important to stress at this moment is that possible tension between these two values (2) and (3) can be attributed to new physics: F_π, determined from the weak decay of the π^+ assumed the validity of the standard model. A possible deviation from it via right-handed currents was opened in [8].

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0920-5632/$ – see front matter © 2012 Published by Elsevier B.V.
doi:10.1016/j.nuclphysbps.2012.12.036

3. Electromagnetic formfactor of charged pion

Vector or more precisely electromagnetic formfactor of the charged pion, F_V^π is defined by

$$\langle \pi^+(p_f)|j_\mu^{\rm elm}|\pi^+(p_i)\rangle = (p_f + p_i)_\mu F_V^\pi[(p_f - p_i)^2]\,. \quad (4)$$

Its calculation within ChPT up to two-loop level can be found in [9] and using dispersive treatment in [10]. Data within the validity of ChPT were taken so far mainly from a space like region (cf.[9]). New measurements in a time-like region almost down to the di-pion threshold (at KLOE10 [11]) urge us to answer the question of validity of ChPT more precisely. For this we will turn to the calculations of the leading logarithms.

Leading logarithms (LL) are logarithms with highest possible power at the given order. Similarly as in the renormalizable theory they can be calculated using only one-loop diagrams [12]. In the renormalizable theory their summation has an important phenomenological consequence: the running coupling constant. In effective theory, as ChPT, the LL coefficients are given only by the form of the leading-order Lagrangian. They are thus parameter-free and without further knowledge of low-energy constants can be used as a rough estimate of the given order. However, the general method for their summation is not known and thus at the moment we must rely only on some simplify cases where it was possible. LL were calculated up to the fifth order in the massive $O(N)$ model (for $N = 3$ it is equivalent to two-flavour ChPT) in [13]. In the massless and large N limit it is indeed possible to resum all LL, the closed form is (cf. also [14], but mind the sign)

$$F_V^{0NLN}(t) = 1 + \frac{1}{N} + \frac{4}{K_t N^2}\Big[1 - \Big(1 + \frac{2}{K_t N}\Big)\log\Big(1 + \frac{K_t N}{2}\Big)\Big], \quad (5)$$

with

$$K_t \equiv \frac{t}{16\pi^2 F^2}\log\Big(-\frac{\mu^2}{t}\Big). \quad (6)$$

The calculated LL together with the resummed function is depicted in Fig. 1. It is clear that convergence in the time-like region is already problematic not far above the threshold. It also shows how important is a resum function (at least in the studied limits). Let us note that even LL are very important in studying the convergence, the actual numerical value is still dominated by the large higher-order coefficients [9].

4. Scalar formfactor

The definition reads

$$F_S^\pi(t \equiv (p-q)^2) = \langle \pi^0(q)|\bar{u}u + \bar{d}d|\pi^0(p)\rangle \quad (7)$$

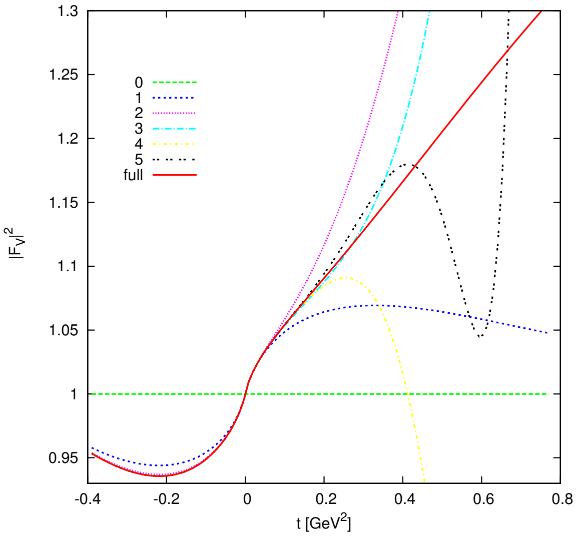

Figure 1: The leading-logarithms for normalized F_V in massless and large N limit in $O(N = 3)$ model.

Its calculation within ChPT exists up to next-to-next-to-leading order in [15]. The fact that this quantity cannot be practically measured today can appear as a problem. However, using $\pi\pi$ phase shifts and the dispersive treatment [16] we can study its energy dependence. Writing

$$F_S^\pi(t) = F_S^\pi(0)\Big(1 + \frac{1}{6}\langle r^2\rangle_S^\pi t + c_S^\pi t^2 + \ldots\Big), \quad (8)$$

one would obtain

$$\langle r^2\rangle_S^\pi = 0.61 \pm 0.04\ {\rm fm}^2, \qquad c_S^\pi = 11 \pm 2\ {\rm GeV}^{-4}\,. \quad (9)$$

These values were recently used in the new global fit of low energy constants of the 3-flavour ChPT [17].

5. Radiative pion decay

The pion decay $\pi^+ \to e^+\nu\gamma$ (see e.g. works in [18]) is interesting in the context of the QCD formfactors because its structure dependent part dominates over the inner Bremsstrahlung due to the helicity suppression. The structure dependent part, connected to the pionic structure, can be further decomposed to the vectorial ($\sim F_V$) and axial ($\sim F_A$) part. Beyond standard model one can consider also tensor radiation part ($\sim F_T$). As there is no significant hint from the recent measurements – the most precise limits are $F_T = (-0.6 \pm 2.8) \times 10^{-4}$ set by the PIBETA group [19] – we will not consider it here. The vector part of the $V - A$ structure, defined as ($e = 1$ for simplicity)

$$\int d^4x e^{iq.x}\langle 0|T(j_\mu^{\rm elm}(x) j_\nu^{V;1-i2}(0)|\pi^+(p)\rangle = \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta \frac{F_V}{m_{\pi^+}}$$