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Oscillations in the K^0 mesons system at S (strangeness) and CP violations

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Abstract

This work considers the $K^o, \bar{K}^o$ meson mixings and the oscillations via K_1^o, K_2^o meson states at strangeness violation by the weak interactions and the K_1^o, K_2^o meson mixings and oscillations via K_S, K_L meson states at CP violation by the weak interactions with taking into account decay widths. We worked in the framework of the masses mixing scheme. It is shown that K_1^o (K_S) meson states appear at big distances from the K^0 mesons source after their decays $(\tau_2 \gg \tau_1)$ due to oscillations of residual K_2^o (K_L) mesons and then again one can see short living K_1^o (K_S) mesons. It is implied that $K_L \leftrightarrow K_S$ meson oscillations are absent. The case when at CP violation the unitarity is violated is considered. The general expressions for the probabilities of meson oscillations (transitions) are given.

Keywords: mesons, oscillations of mesons, mixings, strangeness violation, CP violation, transition probabilities

1. Introduction

The oscillations of K^o mesons (i.e., $K^o \leftrightarrow \bar{K}^o$) were theoretically [1] and experimentally [2] investigated in 50-th and 60-th years. In recent time it was achieved understanding that these processes go as double-stadium process [3–6]. CP violation was detected in 1964 and was reported in work [7]. The detailed study of K^o meson mixing and oscillations are very important since the theory of neutrino oscillations is built in analogy with the theory of K^o meson oscillations.

In the literature [8, 9] a non unitary transformation is used to obtain the K_S, K_L states. It is supposed that these states arise at CP violation. The expression for these states has the following form

$$K_S = (K_1^o + \varepsilon_1 K_2^o)/\sqrt{1+|\varepsilon_1|^2},$$
$$K_L = (K_2^o + \varepsilon_1 K_1^o)/\sqrt{1+|\varepsilon_1|^2}, \tag{1}$$

and for the inverse expressions

$$K_1^o = (K_S - \varepsilon_1 K_L)\frac{\sqrt{1+|\varepsilon_1|^2}}{1-\varepsilon_1^2},$$
$$K_2^o = (K_L - \varepsilon_1 K_S)\frac{\sqrt{1+|\varepsilon_1|^2}}{1-\varepsilon_1^2}. \tag{2}$$

If the wave functions of K_L, K_S mesons are written as

$$K_S = \frac{1-\varepsilon_1}{\sqrt{2(1+|\varepsilon_1|^2)}} e^{-im_S t - \frac{\Gamma_S t}{2}},$$
$$K_L = \frac{1-\varepsilon_1}{\sqrt{2(1+|\varepsilon_1|^2)}} e^{-im_L t - \frac{\Gamma_L t}{2}}, \tag{3}$$

then they are inserted in Eq. (2), one obtains ($\hbar = 1$) for absolute value squared of the first term in (2)

$$|K_1^o|^2 = \frac{|1-\varepsilon_1|^2}{2(1-|\varepsilon_1|^2)}(e^{-\Gamma_S t} + |\epsilon|^2 e^{-\Gamma_L t} - 2|\epsilon| e^{-\frac{(\Gamma_S+\Gamma_L)t}{2}})$$
$$\times \cos((m_L - m_S)t). \tag{4}$$

In Eq. (4) a cross term appears which is responsible for the oscillations. This term can be interpreted as the oscillations between K_S, K_L states, i.e., these states are nonorthogonal ones. It is clear that in this case K_S, K_L states are already not eigenstates with definite masses.

In the framework of quantum mechanics if the states are wave vectors then Eq. (3) have to be written in the following form

$$K_S(t) = (1-\varepsilon_1)/\sqrt{2(1+|\varepsilon_1|^2)} e^{-im_S t - \frac{\Gamma_S t}{2}} K_S(0)$$
$$K_L(t) = (1-\varepsilon_1)/\sqrt{2(1+|\varepsilon_1|^2)} e^{-im_L t - \frac{\Gamma_L t}{2}} K_L(0), \tag{5}$$

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then for the absolute value squared on gets

$$|K_1^o|^2 = (|1-\varepsilon_1|^2)/2(1-|\varepsilon_1|^2)\left(e^{-\Gamma_S t} + |\epsilon|^2 e^{-\Gamma_L t}\right). \quad (6)$$

In Eq. (6) the interference term is absent, i.e., the oscillations are absent.

Now we have to solve the problem: how do oscillations arise in the quantum mechanics approach and how do short living mesons appear at long distances from K^o source?

Let us come to the consideration of $K^o, \bar{K}^o$ mesons oscillations at strangeness violation and K_1^o, K_2^o mesons oscillations at CP violation with taking into account the decay widths by using the standard method from [10]. The value for K_S, K_L (or K_1^o, K_2^o) meson mass difference was measured in the work [11] (the latest value for $m_{K_L} - m_{K_S}$ see in [12]).

2. Vacuum mixings and oscillations of $K^o, \bar{K}^o$ mesons at strangeness violation by the weak interactions with taking into account decay widths

Let us consider the vacuum mixings and oscillations of K^o, $\bar{K}^o$ mesons in the case when width decays are taking into account.

2.1. $K^o, \bar{K}^o$ vacuum mixings

K^o-, $\bar{K}^o$ meson states are produced by strong interaction (i.e, they are eigenstates of this interaction) so mass matrix of K^o mesons has diagonal form [3–6]. Following the traditions for K^o meson mixings and oscillations we will consider the mass matrix with masses in linear form but not in quadratic form, then the mass matrix has the following form

$$\begin{pmatrix} m_{K^oK^o} & 0 \\ 0 & m_{\bar{K}^o\bar{K}^o} \end{pmatrix}. \quad (7)$$

Because of the existence of the weak interactions violating the strangeness ($s \leftrightarrow d$) the mass matrix (7) becomes nondiagonal matrix

$$\begin{pmatrix} m_{K^oK^o} & m_{K^o\bar{K}^o} \\ m_{\bar{K}^oK^o} & m_{\bar{K}^o\bar{K}^o} \end{pmatrix} \to U^-1 \begin{pmatrix} m_{K_1^o} & 0 \\ 0 & m_{K_2^o} \end{pmatrix} U. \quad (8)$$

For obtaining the eigenstates of the weak interactions which violate the strangeness we have to diagonalize this matrix by turning on angle θ. By using this procedure one gets

$$U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad \tan 2\theta = \frac{2m_{K^o\bar{K}^o}}{|\, m_{K^o} - m_{\bar{K}^o}\,|}, \quad (9)$$

$$\sin^2 2\theta = \frac{(2m_{K^o\bar{K}^o})^2}{(m_{K^o} - m_{\bar{K}^o})^2 + (2m_{K^o\bar{K}^o})^2}, \quad \begin{pmatrix} m_{K_1^o} & 0 \\ 0 & m_{K_2^o} \end{pmatrix},$$

$$m_{1,2} = m_{K_1,K_2} = \frac{1}{2}\Big[(m_{K^o} + m_{\bar{K}^o}) \pm \left((m_{K^o} - m_{\bar{K}^o})^2 + 4m^2_{K^o\bar{K}^o}\right)^{1/2}\Big], \quad (10)$$

where K_1^o and K_2^o states are eigenstates of the weak interaction violating strangeness. Now these states are superposition of $K^o, \bar{K}^o$ meson states

$$K_1^o = \cos\theta K^o - \sin\theta \bar{K}^o, \;\; K_2^o = \sin\theta K^o + \cos\theta \bar{K}^o, \quad (11)$$

and inverse transformation gives

$$K^o = \cos\theta K_1^o + \sin\theta K_2^o, \;\; \bar{K}^o = -\sin\theta K_1^o + \cos\theta \bar{K}_2^o. \quad (12)$$

Since $m_{K^oK^o} = m_{\bar{K}^o\bar{K}^o}$ for CPT invariance of the weak interactions the mixing angle θ will be equal to $\frac{\pi}{4}$. Then from Eq. (11) and (12) one gets

$$K_1^o = (K^o - \bar{K}^o)/\sqrt{2}, \quad K_2^o = (K^o + \bar{K}^o)/\sqrt{2}, \quad (13)$$

and $K^o = (K_1^o + K_2^o)/\sqrt{2}$, $\bar{K}^o = (K_1^o - K_2^o)/\sqrt{2}$.

It is necessary to remark that CP parity of K_1^o meson is positive ($CPK_1^o = K_1^o$) and it can decay on two π mesons but CP parity of K_2^o meson is negative ($CPK_2^o = -K_2^o$) and it can decay on three π mesons.

The computation of nondiagonal terms of mass matrix (8)–(10) can be fulfilled by using of quark box Feynman diagrams with $\Delta S = 2$ in the framework of the Standard Model of electroweak interactions [9, 13] with Kabibbo-Kobayashi-Maskawa matrices [14].

2.2. The vacuum K^o meson oscillations with taking into account widths of K_1^o, K_2^o meson decays

Since K_1^o, K_2^o have widths Γ_1, Γ_2 they will decay. We can take into account it in usual manner [10]. K_1^o, K_2^o mesons with masses m_1, m_2 evolve in dependence of time according to the following law

$$K_1^o(t) = e^{-iE_1 t - \frac{\Gamma_1 t}{2}} K_1^o(0), \quad K_2^o(t) = e^{-iE_2 t - \frac{\Gamma_2 t}{2}} K_2^o(0), \quad (14)$$

Using expressions (10) for masses of K_1^o, K_2^o mesons one obtains

$$m_{K_1^o} = m_{K^o} - \Delta, m_{K_2^o} = m_{K^o} + \Delta, \quad (15)$$

where $\Delta = 2m_{K^o\bar{K}^o}$. Since $m_{K^o} \gg \Delta$, then

$$E_1 = \sqrt{p^2 + m^2_{K_1^o}} \cong E_{K^o}\Big(1 - \frac{m_{K^o}\Delta}{E^2_{K^o}}\Big),$$

$$E_2 = \sqrt{p^2 + m^2_{K_2^o}} \cong E_{K^o}\Big(1 + \frac{m_{K^o}\Delta}{E^2_{K^o}}\Big), \quad (16)$$