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A non-local fracture model for composite laminates and numerical simulations by using the FFT method

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ABSTRACT

In this paper, we present a fracture model for composite laminates and its numerical solution by using the Fast Fourier Transforms (FFTs). The FFT-based formulation initially proposed for seeking the average behaviour of linear and non-linear composites by means of the homogenisation procedures [1,2] was adapted to evaluate the damage growth in brittle materials. A non-local damage model based on the maximal principal stress criterion was proposed to assess the failure in the matrix and the fibres. This non-local model was then connected to the Griffith–Irwin criterion in the aim of predicting crack growth. In order to assess the matrix/fibre interface delamination, we have adapted the cohesive model developed by Li [3] for accounting the mixed-mode dependent interface failure. To this end, the interfaces between the matrix and the fibres are replaced by a thin layer of interphase with the purpose of facilitating the FFT simulations. By using the proposed model, we carried out several numerical simulations on fracture process in different specimens. From these studies, we can conclude that the present FFT-based analysis is capable to deal with crack initiation and crack growth in composite laminates with high accuracy and efficiency, especially in the cases of matrix/fibre interface debonding and of multi-crack growth.

1. Introduction

Currently, the carbon fibre reinforced polymer matrix composite laminates have been increasingly employed in various fields including the airplane, hoses in fluid transport, electricity systems or sportive equipments due to their outstanding performance, combining high stiffness and strength at low density, high-specific energy absorption, excellent fatigue performance and high corrosion resistance. Since the stiffness and strength of an individual layer are much higher in the fibre direction than in the transverse direction, an appropriate design associating the physical and mechanical properties of materials with the geometry shapes is necessary to profit from the fibre performance.

The stiffness degradation is an important response to the damage and crack evolution of fibre-reinforced composite laminates under monotone or cyclic loads. The failure mechanisms of these composites have a complex nature due to their sophisticate microscopic and/or mesoscopic structures. Generally speaking, two principal failure mechanisms can be distinguished: the first one is the bulk damage in the matrix and the fibres and the second is the interface debonding between them. The most widely used theoretical tools in the assessment of these failure mechanisms are the fracture mechanics and damage mechanics.

Over the past decades, the continuum damage mechanics [4] has been widely used to predict the isotropic/anisotropic damage evolution of composites by introducing a phenomenological damage tensor D relevant to the matrix and fibre failure [5–9]. The damage/plasticity coupling non-linear models have also been developed to describe the interactive effect of the plastic deformation on damage properties [10–12]. Combined to different failure criteria [13–17] on the initial failure of composite laminates with fibre principal orientations, the continuum damage mechanics has proven its efficiency on bulk failure assessment. However, the numerical application of this approach on interface debonding between the matrix and fibres is quite difficult due to its small thickness dimension for which very fine element meshing is required.

On the other hand, there is an evolving trend to develop energy-based failure criteria in the frame of fracture mechanics. Particularly, the cohesive zone conception [18,19] was adapted to describe the crack propagation in composite materials. The delamination

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mechanisms and crack-bridging mechanisms were thoroughly studied by using the cohesive theory [20–25,3]. These cohesive fracture criteria gain insight into the microscopic damage evolution mechanisms of composites. However, numerical convergence problems may arise from cohesive modelling due to assumed softening properties which require to be solved appropriately as it is implemented into a finite element code. This poses a large challenge to the practical application of the cohesive theory in the damage evolution modelling of composite laminates.

Even though multiple numerical techniques were proposed in the literature, the multiscale progressive failure analysis still meets huge difficulties in incorporating discrete fibre/matrix microscopic modelling and failure mechanisms into damage and failure properties of the whole composite laminates. We believe that alternative methods are useful to enrich the numerical tools in the resolution of such a complicated problem.

In this paper, we propose a non-local fracture model resolved by using the FFT (Fast Fourier Transform) method. A macroscopic failure criterion, in occurrence the maximum stress criterion for simplicity, was used to describe the failure of the matrix and the fibres. This criterion was connected to the Griffith criterion in order to enable it to predict the crack initiation and propagation. A debonding model established on the basis of the Dugdale cohesive concept [3] was adapted for describing the matrix/fibre delamination. The main advantage of the FFT method is its high resolution in discretization of the composite structures such that the matrix/fibre interface can be modelled by a thin layer of interphase. Consequently, some complicated mechanisms such as the matrix/fibre debonding, the propagation of multiple cracks or the crack bridging can directly be simulated.

In this paper, we first recall the FFT-based formulation and the non-local damage model in Sections 2 and 3. In Sections 4 and 5, we describe the adaptation of the Griffith criterion to bulk damage and to interface debonding criteria via the non-local approach. After a short explanation of the numerical algorithm in Section 6, we present in Section 7 several numerical simulations in which the accuracy and the efficiency of the proposed method were examined. Some concluding remarks are given in Section 8.

2. The FFT-based formulation

The damage states that develop in brittle or quasi-brittle materials can be evaluated using an extension of an iterative method on the basis of Fast Fourier Transforms (FFT), originally proposed by Moulinec and Suquet [1,2] and Michel et al. [26] for homogenising linear and non-linear composites. The FFT-based formulation for a periodic heterogeneous cell with damage can straightforwardly be written according to the original FFT scheme. By omitting the rigid body motion, the displacement in a periodic cell is split into two parts:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}*(\mathbf{x}) + \mathbf{E}\cdot\mathbf{x} \tag{1}$$

where $\mathbf{x}$ denotes the Cartesian coordinates originated at the geometrical centre of the cell; $\mathbf{u}*(\mathbf{x})$ is the periodically oscillatory part of the displacement with $\int_\Omega \mathbf{u}*(\mathbf{x})dV = \mathbf{0}$; $\mathbf{E}$ is the average strain tensor. In the case when the damage field can be expressed by a scalar variable $D(\mathbf{x}) \in [0,1]$, the problem of elasticity for an inhomogeneous elastic composite under periodic boundary conditions writes:

$$\begin{aligned} &\boldsymbol{\sigma}(\mathbf{x}) = [1 - D(\mathbf{x}) + k_0]\mathbf{C}(\mathbf{x}) : [\boldsymbol{\varepsilon}(\mathbf{u}*(\mathbf{x})) + \mathbf{E}] \\ &div\boldsymbol{\sigma}(\mathbf{x}) = 0 \\ &\mathbf{u}*(\mathbf{x}) \text{ is periodic } \forall \mathbf{x} \in \Omega, \boldsymbol{\sigma}.\mathbf{n} \text{ is anti} - \text{periodic } \forall \mathbf{x} \in \partial\Omega \end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress field; $\varepsilon(\mathbf{u}*)$ is the fluctuation term of the strain field in Ω and satisfies $\int_\Omega \boldsymbol{\varepsilon}(\mathbf{u}*(\mathbf{x}))dV = \mathbf{0}$; the damage field $D(\mathbf{x})$ and the stiffness tensor $\mathbf{C}(\mathbf{x})$ are also periodic; k_0 is a parameter of very small value, it ensures the existence and uniqueness of the numerical solution; $\mathbf{n}$ is the unit outward vector of the cell boundary. This local problem can be resolved by introducing a polarisation stress field $\boldsymbol{\tau}(\mathbf{x})$,

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}^r : \boldsymbol{\varepsilon}(\mathbf{u}*) + \boldsymbol{\tau}(\mathbf{x}) \tag{3}$$

with $\mathbf{C}^r$ being the stiffness of a homogeneous reference material and

$$\boldsymbol{\tau}(\mathbf{x}) = (1 - D + k_0)\mathbf{C} : [\boldsymbol{\varepsilon}(\mathbf{u}*) + \mathbf{E}] - \mathbf{C}^r : \boldsymbol{\varepsilon}(\mathbf{u}*) \tag{4}$$

The solution of (2) can be expressed by means of the periodic Green operator $\boldsymbol{\Gamma}$ associated with $\mathbf{C}^r$, namely

$$\boldsymbol{\varepsilon}(\mathbf{x}) = -\int_\Omega \boldsymbol{\Gamma}(\mathbf{x} - \mathbf{y}) : \boldsymbol{\tau}(\mathbf{y})d\mathbf{y} \tag{5}$$

By performing the Fourier transformation, this convolution integral is transformed into a direct tensor product:

$$\hat{\boldsymbol{\varepsilon}}(\boldsymbol{\xi}) = -\widehat{\boldsymbol{\Gamma}}(\boldsymbol{\xi}) : \hat{\boldsymbol{\tau}}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \neq 0 \quad \hat{\boldsymbol{\varepsilon}}(0) = \mathbf{E} \tag{6}$$

where $\hat{\boldsymbol{\varepsilon}}$, $\hat{\boldsymbol{\tau}}$ and $\widehat{\boldsymbol{\Gamma}}$ are respectively the Fourier transforms of $\boldsymbol{\varepsilon}$, $\boldsymbol{\tau}$ and $\widehat{\boldsymbol{\Gamma}}$, $\boldsymbol{\xi}$ denotes the frequencies in Fourier space. When the reference material is isotropic with the Lamé coefficients λ^r and μ^r, the Green operator $\widehat{\boldsymbol{\Gamma}}$ takes the form:

$$\begin{aligned} \hat{\Gamma}_{ijkl} = &\frac{1}{4\mu^r|\xi|^2}\left(\delta_{ki}\xi_k\xi_l + \delta_{li}\xi_k\xi_j + \delta_{kj}\xi_l\xi_i + \delta_{lj}\xi_k\xi_i\right) \\ &- \frac{(\lambda^r + \mu^r)}{\mu^r(\lambda^r + 2\mu^r)}\frac{\xi_i\xi_j\xi_k\xi_l}{|\xi|^4} \end{aligned} \tag{7}$$

The anti-transforming of (6) gives the strain field of the problem. However, since the polarisation stress field is a priori unknown for a damaged heterogeneous material, iterative procedure has to be used to obtain a compatible strain field and then a stress field in equilibrium.

In the present work, only the plane problems are considered. For plane strain, all the precedent formulas are directly valid for bi-dimensional simulations. For plane stress, the same equations in plane strain can be kept if the Lamé coefficient λ is replaced by $\lambda\frac{2\mu}{\lambda+2\mu}$.

3. Non-local damage model

Numerous continuum damage models exist in the literature to describe the progressive degradation of materials. The accuracy of the classical damage models often depend on the finite element discretization in their numerical implementation. In order to overcome this shortcoming, various regularization methods have been proposed. Among these methods, the so-called non-local approaches are widely used. The basic idea of this approach consists in replacing the local damage driving force, an effective stress σ_e for example, by its weighted average over a representative volume V [27]:

$$\tilde{\sigma}_e(\mathbf{x}) = \frac{1}{\int_V \alpha(\mathbf{x} - \mathbf{y})d\mathbf{y}}\int_V \alpha(\mathbf{x} - \mathbf{y})\sigma_e(\mathbf{y})d\mathbf{y} \tag{8}$$

In the literature, α is often taken as the Gaussian function. In the present work, a cone-shape weighting function is used in the place of the Gaussian function for its periodic feature required in FFT calculations. This function writes:

$$\alpha(r) = \begin{cases} 0 & r > R \\ 1 - \frac{r}{R} & r \leqslant R \end{cases} \tag{9}$$

where $r = \|\mathbf{x} - \mathbf{y}\|$; R is the radius of non-local action, representing a material characteristic length which defines the size of interaction zone for failure processes.