Physics Letters B 784 (2018) 163–168

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Inflation in gauge theory of gravity with local scaling symmetry and quantum induced symmetry breaking

Yong Tang [a,*], Yue-Liang Wu [b,c,d]

[a] Department of Physics, Faculty of Science, The University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan
[b] International Centre for Theoretical Physics Asia-Pacific, Beijing, China
[c] Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
[d] University of Chinese Academy of Sciences, Beijing 100049, China

ARTICLE INFO

Article history:
Received 29 May 2018
Received in revised form 26 July 2018
Accepted 26 July 2018
Available online 27 July 2018
Editor: J. Hisano

ABSTRACT

Motivated by the gauge theory of gravity with local scaling symmetry proposed recently in [1,2], we investigate whether the scalar field therein can be responsible for the inflation. We show that the classical theory would suffer from the difficulty that inflation can start but will never stop. We explore possible solutions by invoking the symmetry breaking through quantum effects. The effective potential of the scalar field is shown to have phenomenologically interesting forms to give viable inflation models. The predictions of physical observables agree well with current cosmological measurements and can be further tested in future experiments searching for primordial gravitational waves.

1. Introduction

Inflation has been one of the popular paradigms to solve several notable problems in cosmology since 1980s. It provides a basic framework to explain the origins of initial conditions in standard big-bang theory. In the inflationary epoch, a scalar field is usually dominating the energy density of the universe and results in an exponential expansion of the background spacetime. Quantum fluctuation of this scalar field is responsible for the primordial inhomogeneity and anisotropy that will lead to our observed cosmos.

Motivated by the gauge theory of gravity with local scaling symmetry proposed recently in [1,2], we investigate whether the scalar field therein can be responsible for the inflation. In Refs. [1,2], a general hyperunified field theory of gravity is constructed, incorporating the spin gauge group and local scaling symmetry. To make the theory scaling invariant, a fundamental real scalar field ϕ and its corresponding Weyl gauge field W_μ have to be introduced. Another basic field $\chi^a_{\ \mu}$ is related with the traditional metric tensor through $g_{\mu\nu} = \chi^a_{\ \mu}\chi^b_{\ \nu}\eta_{ab}$ where the sign convention $\eta_{ab} = (1, -1, -1, -1)$ is adopted.

Induced gravity can be generally referred as theories where the Planck scale is generated by other fields [3,4]. In the literature, global scaling symmetry has been used in building inflation models (see [5–15] for examples), aiming to introduce no dimensional parameters or explain the hierarchies between different energy scales. As we shall see, whether the scaling symmetry is global or local actually has some important differences. In the presence of a local symmetry, not only a Weyl gauge field has to be accompanying, but also some new term concerning the scalar field appears. Furthermore, the local scaling symmetry indicates the existence of a fundamental energy scale [1,2].

This paper is organized as follows. In Sec. 2 we discuss the theoretical formalism briefly and illustrate how the classical scaling invariant theory would not be able to give viable inflation. Then in Sec. 3 we show how quantum effects can break the scaling symmetry and induce an effective potential so as to provide viable inflation models. The numerical investigation is presented in Sec. 4, with Fig. 3 as the main numeric result. Finally, we give our conclusion.

2. Formalism

The full theory and its formalism has been presented in detail in Refs. [1,2], where the gauge–gravity and gravity–geometry correspondences are explicitly demonstrated to obtain the conformal scaling gauge invariant Einstein–Hilbert action for gravitational interaction with a fundamental scalar field.[1] For our interest in inflation, we may study the most relevant action with the traditional

* Corresponding author.
E-mail address: ytang@hep-th.phys.s.u-tokyo.ac.jp (Y. Tang).

[1] One may also look into the nice introductory review [16] for gauge theory of Einstein's gravity.

https://doi.org/10.1016/j.physletb.2018.07.048

metric tensor $g_{\mu\nu}=\chi^a{}_\mu\chi^b{}_\nu\eta_{ab}$. The action can be written as follows

$$S\equiv\int d^4x\mathcal{L}=\int d^4x\sqrt{-g}\Big[\alpha\left(\phi^2R-6\partial_\mu\phi\partial^\mu\phi\right)+\frac{1}{2}D_\mu\phi D^\mu\phi-\frac{\beta}{4!}\phi^4-\frac{1}{4}W_{\mu\nu}W^{\mu\nu}\Big],\qquad(2.1)$$

where $g\equiv\det\left(g_{\mu\nu}\right)$, α and β are constant parameters that will be decided by observations, and covariant derivative D_μ for real scalar field ϕ is given by $D_\mu\phi=\left(\partial_\mu-g_W W_\mu\right)\phi$ (note that there is no factor i in front of g_W, different from usual $U(1)$ theory), g_W is a coupling associated with Weyl gauge field W_μ. Note that the action is just a subset of the whole action which should include other matter fields (such as fields in standard model) whose effects will be discussed shortly.

We can also explicitly add a term $\delta\mathcal{L}$ that does not share the same symmetries as other terms. As we will show later, $\delta\mathcal{L}$ is actually crucial to realize a realistic inflation. The above theory has no intrinsic energy scale in the Lagrangian except in $\delta\mathcal{L}$. As long as the dimensional parameters in $\delta\mathcal{L}$ are much smaller than the relevant physical scale, such a theory has the classical scaling symmetry which will be broken by quantum effects. We shall discuss more about this point in Sec. 3.

At the moment, let us first ignore $\delta\mathcal{L}$ and focus on the other parts. Each term in the rest of the action is invariant under local conformal scaling symmetry or Weyl symmetry with a positive function $\lambda(x)$:

$$\begin{aligned}g_{\mu\nu}(x)&\to\bar{g}_{\mu\nu}(x)=\lambda^2(x)\,g_{\mu\nu}(x),\\ \phi(x)&\to\bar{\phi}(x)=\lambda^{-1}(x)\,\phi(x),\\ W_\mu(x)&\to\overline{W}_\mu(x)=W_\mu(x)-\frac{1}{g_W}\partial_\mu\ln\lambda(x).\end{aligned}\qquad(2.2)$$

If the above symmetry is exact or when we only consider the classical dynamics, we can easily see that for any generic $g(x)$ and $\phi(x)$ it is possible to make a local transformation, by choosing a proper $\lambda(x)$, to have either $\sqrt{-\bar{g}}=1$ or $\bar{\phi}(x)=\text{const}$, but not simultaneously.[2] If the symmetry was global, W_μ would be absent. Also the second term in parentheses with negative sign $-6\partial_\mu\phi\partial^\mu\phi$ is dropped as in Refs. [6–10].

Before discussing the theory in Eq. (2.1), we shall warm up with the following action

$$S=\int d^4x\sqrt{-g}\left[\alpha\left(\phi^2R-6g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)-\frac{\beta}{4!}\phi^4\right],\qquad(2.3)$$

which still preserves the local conformal symmetry

$$g_{\mu\nu}(x)\to\bar{g}_{\mu\nu}(x)=\lambda^2(x)\,g_{\mu\nu}(x)\,,\phi(x)\to\bar{\phi}(x)=\lambda^{-1}(x)\,\phi(x)\,.\qquad(2.4)$$

At first glance, the above theory seems to add a scalar degree of freedom (dof) to Einstein's general theory of gravity and mimics the scalar–tensor theory. However, if we make a redefinition of metric field

$$\bar{g}_{\mu\nu}(x)=\Omega^2(x)\,g_{\mu\nu}(x)\,,\qquad(2.5)$$

where $\Omega(x)\equiv\sqrt{2\alpha}\phi/M_p$, $M_p^2\equiv1/(8\pi G)$ is the Planck scale. We can rewrite the action with the new field variables

$$\int d^4x\sqrt{-\bar{g}}\,\Omega^{-4}\Big\{\alpha\Big[\phi^2\Omega^2\left(\bar{R}-6\bar{\Box}\ln\Omega+6\bar{g}^{\mu\nu}\partial_\mu\ln\Omega\partial_\nu\ln\Omega\right)-6g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\Big]-\frac{\beta}{4!}\phi^4\Big\}.$$

We have used the relation

$$R=\Omega^2\left[\bar{R}-6\bar{\Box}\ln\Omega+6\bar{g}^{\mu\nu}\partial_\mu\ln\Omega\partial_\nu\ln\Omega\right],$$
$$\bar{\Box}\ln\Omega=\frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu\ln\Omega\right).$$

The second term in the square bracket will eventually lead to a total derivative and vanishes on the surface, and the third term cancels with the derivatives of ϕ. After substituting Ω, we can obtain

$$\begin{aligned}S&=\int d^4x\sqrt{-\bar{g}}\Big\{\Big[\frac{M_p^2}{2}\left(\bar{R}-6\bar{\Box}\ln\Omega+6\bar{g}^{\mu\nu}\partial_\mu\ln\Omega\partial_\nu\ln\Omega\right)-6\alpha\Omega^{-4}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\Big]-\frac{\beta}{4!}\frac{M_p^4}{4\alpha^2}\Big\}\\&=\int d^4x\sqrt{-\bar{g}}\left[\frac{M_p^2}{2}\bar{R}-\frac{\beta}{4!}\frac{M_p^4}{4\alpha^2}\right].\end{aligned}\qquad(2.6)$$

This is exactly the Einstein–Hilbert action with a cosmological constant $\Lambda=\frac{\beta}{4!}\frac{M_p^4}{4\alpha^2}$ that leads an exponential expansion of Universe for $\beta>0$. This also shows that total number of physical dofs does not differ from Einstein's gravity. Extending the discussion into the framework of quantum field theory does not change the above conclusions since the number of dofs does not change from classical to quantum theories. This conclusion is also true even if we break the local scaling symmetry by adding to the potential with terms that depend on ϕ only but not on its derivatives. Adding symmetry-breaking derivative terms would make significant different, as we shall show shortly.

We can get the same result in a different way by choosing $\lambda(x)=\Omega(x)$ in Eq. (2.4)

$$\begin{aligned}\mathcal{L}&=\sqrt{-\bar{g}}\left[\alpha\left(\bar{\phi}^2\bar{R}-6\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi}\right)-\frac{\beta}{4!}\bar{\phi}^4\right]\\&\xrightarrow{\bar{\phi}=\frac{M_p}{\sqrt{2\alpha}}}\sqrt{-\bar{g}}\left[\frac{M_p^2}{2}\bar{R}-\frac{\beta}{4!}\frac{M_p^4}{4\alpha^2}\right],\end{aligned}\qquad(2.7)$$

which effectively chooses a frame $\phi(x)=M_p/\sqrt{2\alpha}$. An interesting observation is that if we choose $\lambda(x)=a^{-1}(x)$ ($a(x)$ is the scale factor in Friedmann–Walker metric), in such a case we would have the action in a flat spacetime,

$$\bar{R}=0\text{ and }S=\int d^4x\left[-6\alpha\eta^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi}-\frac{\beta}{4!}\bar{\phi}^4\right].\qquad(2.8)$$

The equation of motion for $\bar{\phi}$ is

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\bar{\phi}-\frac{\beta}{72\alpha}\bar{\phi}^3=0,\qquad(2.9)$$

which has a non-trivial solution

$$\bar{\phi}(x)=\frac{M}{C\pm x\cdot k},\ \eta^{\mu\nu}k_\mu k_\nu=\frac{\beta M^2}{144\alpha}.\qquad(2.10)$$

[2] Note that $|\bar{g}|=1$ does not always mean flat geometry. According to Weyl–Schoutem theorem, a Riemannian manifold with dimension $n\geq4$ is conformally flat if and only if the Weyl tensor vanishes. Nevertheless, for our interest in inflation, we will always focus on the case where the metric is conformally flat, $g_{\mu\nu}=a^2(x)\,\eta_{\mu\nu}$, where $a(x)$ is the scale factor.