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Black-hole stability in non-local gravity

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ABSTRACT

We prove that Ricci-flat vacuum exact solutions are stable under linear perturbations in a new class of weakly non-local gravitational theories finite at the quantum level.

1. Introduction

A popular motivation to study quantum gravity is that a gravitational interaction governed by the laws of quantum mechanics may avoid spacetime singularities [1]. A class of proposals aiming at such result and that raised in the last few years is known under the umbrella name of non-local quantum gravity (see [1,2] for reviews and references therein). Non-local quantum gravity is a perturbative field theory of gravitation whose dynamics is characterized by form factors (operators in kinetic-like terms) with infinitely many derivatives. The quantum theory has the good taste of being unitary and renormalizable or finite, for certain choices of form factors. However, the fate of classical singularities is still under debate. For instance, the classical theory seems able to resolve the big-bang singularity and replace it with a bounce [3,4] but singular solutions such as a universe filled with radiation are possible [5].

The case of black holes is especially interesting because, currently, there are three different views: black holes in non-local gravity may form but are singular [6,7], they form and are regular [8–15], or they may not even form as asymptotic, classically stable states [16,17]. The first view is the subject of the present paper (we will comment about the second view in the concluding section). Although there is no complete proof of black-hole stability in non-local theories known to be renormalizable, this proof exists for their non-renormalizable counterparts without the Ricci-tensor–Ricci-tensor term [6]. Here we fill this gap and show analytically that black holes may form and be stable in a wide class of theories in the absence of Weyl–Weyl terms,[1] encompassing most of the proposals whose renormalizability has been studied so far [2,22–28]. This result should not be taken as a negative trait of non-local quantum gravity, for the simple reason (among others we will discuss later) that the control of divergences in the quantum theory goes much beyond the naive removal of all *classical* singularities.

In section 2, we introduce a new theory of non-local gravity where the Laplace–Beltrami operator $\Box$ is replaced by the Lichnerowicz operator $\Delta_{\rm L}$. This change does not modify perturbative renormalizability of the theory with respect to previous proposals but it allows one to address the stability issue (section 4) of Ricci-flat solutions (section 3) exactly. An extension of the theory to the electromagnetic sector is discussed in section 5, while section 6 is devoted to conclusions.

Our conventions are the following. The metric tensor $g_{\mu\nu}$ has signature $(-+\cdots+)$ and the curvature tensors are defined as $R^{\mu}{}_{\nu\rho\sigma}=-\partial_\sigma\Gamma^{\mu}_{\nu\rho}+\dots$, $R_{\mu\nu}=R^{\rho}{}_{\mu\rho\nu}$ and $R=g^{\mu\nu}R_{\mu\nu}$. Terms quadratic in the Ricci tensor or scalar but not in the Riemann tensor will be denoted as $O(\mathbf{Ric}^2)$.

2. A new class of non-local gravity theories

A general class of theories compatible with unitarity and super-renormalizability or finiteness has the following structure in D dimensions:

[1] In general, a Weyl–Weyl term $C_{\mu\nu\sigma\tau}\mathcal{F}_3(\Box)C^{\mu\nu\sigma\tau}$ in the action [18,19] may restrict the theory further. The space of solutions is shrunk [20] with respect to its span in the absence of this extra term [21] and, in particular, Schwarzschild singular solutions are forbidden there [15].

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$$S=\frac{1}{2\kappa^2}\int d^Dx\sqrt{|g|}\left[R+R\mathcal{F}_0(\Delta_{\rm L})R+R_{\mu\nu}\mathcal{F}_2(\Delta_{\rm L})R^{\mu\nu}+V_g\right], \tag{1}$$

where the 'potential' term V_g is at least cubic in the curvature and at least quadratic in the Ricci tensor and $\mathcal{F}_{0,2}$ are form factors, functions of the Lichnerowicz operator $\Delta_{\rm L}$. When acting on a rank-2 symmetric tensor,

$$\begin{aligned}\Delta_{\rm L}X_{\mu\nu}&=2R^\sigma{}_{\mu\nu\tau}X^\tau{}_\sigma+R_{\mu\sigma}X^\sigma{}_\nu+R_{\sigma\nu}X^\sigma{}_\mu-\Box X_{\mu\nu}\\&=-2R_{\mu\sigma\nu\tau}X^{\sigma\tau}+R_{\mu\sigma}X^\sigma{}_\nu+R_{\sigma\nu}X^\sigma{}_\mu-\Box X_{\mu\nu}\,.\end{aligned} \tag{2}$$

On the trace X^μ_μ or on a scalar X, $\Delta_{\rm L}X=-\Box X$.

The Lagrangian (1) is proposed here for the first time but it has all the same properties of the theories considered in [2,22–28], in particular, perturbative unitarity and finiteness. The main difference with respect to previous literature is that $\mathcal{F}_{0,2}$ are functions of $\Delta_{\rm L}$ instead of the Laplace–Beltrami operator $\Box$. On a flat background, these operators coincide. Tree-level unitarity is not affected by the Riemann and Ricci tensors present in the form factors because, when we expand the action to second order in the graviton perturbation, such tensors do not give contributions to the propagator. Indeed, the form factors are inserted between two Ricci tensors or scalars that are already linear in the graviton around Minkowski space. Therefore, the Lichnerowicz operator can only affect vertices in Feynman diagrams, but the power-counting analysis of [23,24] still holds. One has only to replace the variation of the $\Box$ operator with the variation of $\Delta_{\rm L}$.

3. Exact Ricci-flat solution

Let us recall the proof that any Ricci-flat spacetime (Schwarzschild, Kerr, and so on) is an exact solution in a large class of super-renormalizable or finite gravitational theories at least quadratic in the Ricci tensor [5]. This calculation was done with the Laplace–Beltrami operator $\Box$ but we adapt it to the $\Box\to-\Delta_{\rm L}$ case straightforwardly.

The equations of motion (EOM) in a compact notation [29] for the action (1) read

$$\begin{aligned}E_{\mu\nu}:=&\frac{\delta\left[\sqrt{|g|}\left(R+R\mathcal{F}_0(\Delta_{\rm L})R+R_{\alpha\beta}\mathcal{F}_2(\Delta_{\rm L})R^{\alpha\beta}+V_g\right)\right]}{\sqrt{|g|}\delta g^{\mu\nu}}\\
=&\,G_{\mu\nu}-\frac12 g_{\mu\nu}R\mathcal{F}_0(\Delta_{\rm L})R-\frac12 g_{\mu\nu}R_{\alpha\beta}\mathcal{F}_2(\Delta_{\rm L})R^{\alpha\beta}\\
&+2\frac{\delta R}{\delta g^{\mu\nu}}\mathcal{F}_0(\Delta_{\rm L})R+\frac{\delta R_{\alpha\beta}}{\delta g^{\mu\nu}}\mathcal{F}_2(\Delta_{\rm L})R^{\alpha\beta}\\
&+\frac{\delta R^{\alpha\beta}}{\delta g^{\mu\nu}}\mathcal{F}_2(\Delta_{\rm L})R_{\alpha\beta}+\frac{\delta\Delta^r_{\rm L}}{\delta g^{\mu\nu}}\left[\frac{\mathcal{F}_0(\Delta^l_{\rm L})-\mathcal{F}_0(\Delta^r_{\rm L})}{\Delta^r_{\rm L}-\Delta^l_{\rm L}}RR\right]\\
&+\frac{\delta\Delta^r_{\rm L}}{\delta g^{\mu\nu}}\left[\frac{\mathcal{F}_2(\Delta^l_{\rm L})-\mathcal{F}_2(\Delta^r_{\rm L})}{\Delta^r_{\rm L}-\Delta^l_{\rm L}}R_{\alpha\beta}R^{\alpha\beta}\right]+\frac{\delta V_g}{\delta g^{\mu\nu}}=0\,,\end{aligned} \tag{3}$$

where $\Delta^{l,r}_{\rm L}$ act on, respectively, the left and right arguments (on the right of the incremental ratio) inside the brackets.

Replacing the *Ansatz* $R_{\mu\nu}=0$ in the equations of motion (3), the following chain of implications holds in vacuum:

$$R_{\mu\nu}=0\quad\Longrightarrow\quad E_{\mu\nu}=0\quad\Longleftrightarrow\frac{\delta V_g}{\delta g^{\mu\nu}}=O(\mathbf{Ric})\,. \tag{4}$$

In particular, the Schwarzschild metric, the Kerr metric and all the known Ricci-flat metrics in vacuum Einstein gravity are exact solutions of the non-local theory.

4. Stability

In this section, we study the stability of Ricci-flat solutions under linear perturbations. We focus on the minimal finite theory of gravity compatible with super-renormalizability. Namely, we select $\mathcal{F}_0=-\mathcal{F}_2/2$ (we also redefine $\mathcal{F}_2\equiv\gamma$) in (1) and (3). Tree-level unitarity requires

$$\gamma(\Delta_{\rm L})=\frac{e^{H(\Delta_{\rm L})}-1}{-\Delta_{\rm L}}\,, \tag{5}$$

where H is an analytic function that can be expanded in a series with infinite convergence radius. This type of 'gentle' non-locality is called weak and is discussed elsewhere. We will not make use of the form factor (5) until later.

At quadratic order in the Ricci tensor, the EOM read

$$G_{\mu\nu}+2\frac{\delta R_{\alpha\beta}}{\delta g^{\mu\nu}}\gamma(\Delta_{\rm L})G^{\alpha\beta}+O(\mathbf{Ric}^2)=0\,. \tag{6}$$

Notice that the omitted higher-order curvature term is at least quadratic in the Ricci tensor but not in the Riemann tensor. This property is crucial for the proof of stability.

To this purpose, we only need to keep terms at most linear in the Ricci tensor in the EOM. Using the variation

$$\frac{\delta R_{\alpha\beta}}{\delta g^{\mu\nu}}=\frac12 g_{\alpha(\mu}g_{\nu)\beta}\Box+\frac12 g_{\mu\nu}\nabla_\alpha\nabla_\beta-g_{\alpha(\mu|}\nabla_\beta\nabla_{|\nu)}\,,$$

we can rewrite (6) as

$$\begin{aligned}0=&\,G_{\mu\nu}+2\Big[\frac12 g_{\alpha(\mu}g_{\nu)\beta}\Box+\frac12 g_{\mu\nu}\nabla_\alpha\nabla_\beta\\
&-\frac12\left(g_{\alpha\mu}\nabla_\beta\nabla_\nu+g_{\alpha\nu}\nabla_\beta\nabla_\mu\right)\Big]\gamma(\Delta_{\rm L})G^{\alpha\beta}+O(\mathbf{Ric^2})\\
=&\,G_{\mu\nu}+\Box\gamma(\Delta_{\rm L})G_{\mu\nu}\underbrace{+g_{\mu\nu}\nabla_\alpha\nabla_\beta\gamma(\Delta_{\rm L})G^{\alpha\beta}}_{①}\\
&\underbrace{-2\nabla_\beta\nabla_{(\mu}\gamma(\Delta_{\rm L})G_{\nu)}{}^\beta}_{②}+O(\mathbf{Ric^2})\,.\end{aligned} \tag{7}$$

In the appendix we prove the following non-trivial identity up to Ricci-square terms:

$$\nabla^\mu\left[\gamma(\Delta_{\rm L})G_{\mu\nu}\right]=O(\mathbf{Ric^2})\,. \tag{8}$$

Using this expression, one immediately finds that

$$①=O(\mathbf{Ric^2})\,.$$

Also, the commutator of covariant derivatives on a symmetric tensor is

$$[\nabla_\beta,\nabla_\mu]X^{\alpha\beta}=R^\alpha{}_{\lambda\beta\mu}X^{\lambda\beta}+R_{\lambda\mu}X^{\lambda\alpha}\,. \tag{9}$$

Plugging (8) and (9) with $X^{\alpha\beta}=\gamma(\Delta_{\rm L})G^{\alpha\beta}$ (which is linear in the Ricci tensor) into ②, up to operators quadratic in the Ricci tensor one has

$$\begin{aligned}②=&-\left(g_{\alpha\mu}\nabla_\beta\nabla_\nu+g_{\alpha\nu}\nabla_\beta\nabla_\mu\right)\gamma(\Delta_{\rm L})G^{\alpha\beta}\\
=&-\left(g_{\alpha\mu}\nabla_\nu\nabla_\beta+g_{\alpha\mu}[\nabla_\beta,\nabla_\nu]+g_{\alpha\nu}\nabla_\mu\nabla_\beta+g_{\beta\mu}[\nabla_\beta,\nabla_\mu]\right)\\
&\times\gamma(\Delta_{\rm L})G^{\alpha\beta}\\
=&-g_{\alpha\mu}R^\alpha{}_{\lambda\beta\nu}X^{\lambda\beta}-g_{\alpha\nu}R^\alpha{}_{\lambda\beta\mu}X^{\lambda\beta}+O(\mathbf{Ric^2})\\
=&-R_{\mu\lambda\beta\nu}X^{\lambda\beta}-R_{\nu\lambda\beta\mu}X^{\lambda\beta}+O(\mathbf{Ric^2})\\
=&\,2R_{\mu\beta\nu\lambda}X^{\beta\lambda}+O(\mathbf{Ric^2})\,.\end{aligned}$$