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Implications of the generalized entropy formalisms on the Newtonian gravity and dynamics

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ABSTRACT

Employing the Verlinde's hypothesis, and considering two well-known generalized entropy formalisms, two modifications to the Newtonian gravity are derived. In addition, it has been shown that the generalized entropy measures may also provide theoretical basis for the Modified Newtonian Dynamics (MOND) theory and generate its modified forms. Since these entropy measures are also successful in describing the current accelerated universe, our results indicate that the origin of dark sectors of cosmos may be unified to meeting the generalized entropy measures instead of the Boltzmann–Gibbs entropy by the gravitational systems due to the long-range nature of gravity.

1. Introduction

The correspondence between the first law of thermodynamics on the boundary of spacetime and the field equations of gravity, describing the system evolution in bulk [1–4], suggests a profound connection between the laws of thermodynamics and gravity which further supports the holography proposal [5–9]. Based on the Verlinde's hypothesis [7], the tendency of systems to increase their entropy leads to the emergence of gravity between the holographic screens. It is important to note that the entropy expression plays a crucial role in this theory. In fact, diverse corrections to the entropy-area relation presents various modifications to the gravitational theories and their corresponding cosmology [10–28]. It has also been shown [23–27] that if the Unruh temperature [29] is attributed to the holographic screen, then the quantum statistics of surface degrees of freedom may help us in obtaining a theoretical basis for the Modified Newtonian Dynamics (MOND) theory [30–32]. It is also worthy to note here that the quantum aspects of surface degrees of freedom imply that a_0, appeared in MOND theory, depends on the energy content of system [14,23–27].

One property of gravity is its long-range nature which motivates physicists to use generalized entropy formalisms [33–39] in order to study various gravitational and cosmological phenomena [21,22,40–57]. In the generalized entropy formalisms, systems are described by the power-law distributions of probabilities (P_i^q) instead of the ordinary linear distribution (P_i) [33–39], and the additional new free parameters, such as q, can be evaluated by fitting with data [33–39]. The Tsallis generalized entropy [33,34], an one-free parameter measure, can be combined with the Verlinde's hypothesis [7] to obtain a MOND theory [47]. Additionally, using the Verlinde approach, it has also been shown that the power-law and logarithmic corrections to the Bekenstein entropy lead to modified versions for the MOND theory [14]. These attempts motivate us to study relations between those generalized entropies which are successful in describing cosmological and gravitational phenomena [21,22,40–57] and various MOND theories.

In the present Letter, by taking the various generalized entropy formalisms as well as the entropic origin of gravity into account, we are going to derive some MOND theories and the implications of these generalized entropy measures on the Newtonian gravity. We are focusing on those the generalized entropy measures successful in describing the current accelerated universe [52–57]. The paper is organized as follows. Some generalized entropy measures and the general relation between the system entropy and the gravitational force in the Verlinde approach have been shown in the

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next section. In sec. III, we address some MOND theories and corrected Newtonian gravities based on the generalized entropies. The last section is devoted to a summary. For the sake of simplicity, we set $G=\hbar=c=k_B=1$, where k_B is the Boltzmann constant, throughout the article.

2. Generalized entropy formalism, Verlinde hypothesis and the gravitational force

For a system with W discrete states, where each state has probability P_i, Tsallis and Rényi entropies are defined as

$$S_T=\frac{1}{1-q}\sum_{i=1}^{W}\left(P_i^q-P_i\right), \tag{1}$$

and

$$\mathcal{S}=\frac{1}{1-q}\ln\sum_{i=1}^{W}P_i^q, \tag{2}$$

respectively, where q is a free parameter [35]. Using the fact that $\sum P_i=1$, we can combine these equations to arrive at

$$(1-q)S_T+1=e^{(1-q)\mathcal{S}}, \tag{3}$$

which finally leads to [41]

$$\mathcal{S}=\frac{1}{\delta}\ln(1+\delta S_T), \tag{4}$$

where we have use the $\delta=1-q$ expression. There is also a two-parametric generalized entropy which is called the Sharma-Mittal entropy and is written as [39,55]

$$S_{SM}=\frac{1}{\alpha}\left((1+\delta S_T)^{\frac{\alpha}{\delta}}-1\right), \tag{5}$$

where $\alpha\equiv 1-r$, and r is a new free parameter. Some cosmic applications of $\mathcal{S}$ and S_{SM} can be found in Refs. [53–56]. In general, the free parameters r and q should be evaluated by comparing the theory with the observations [39], meaning that the free parameters are not the same for all systems. This is in full agrement with gravitational and cosmological studies [22,40–57].

In addition, we assume that system has a boundary with area A, and consists N degrees of freedom which satisfy the energy equipartition law [47,53]

$$E=M=\frac{NT}{2}. \tag{6}$$

Here, T is the boundary temperature and M denotes the mass content of system. Moreover, in our unit, A and N are in a mutual relation as [7,15]

$$A=N, \tag{7}$$

claiming that the area change per unit change of information is one (or equally $\Delta A=\Delta N=1$) [7,15]. It was argued that the Bekenstein's entropy expression ($S_B=A/4$), the entropy of a system with boundary A [58], is a proper candidate for S_T [41,42,53], a result confirmed by using the Tsallis formalism to evaluate the black hole entropy in loop quantum gravity [49]. Thus, replacing S_B with S_T in the above generalized entropy measures, one can easily find the entropy-area relation in generalized entropy formalism [53–56].

Based on the entropic force scenario, the absolute value of gravitational force applied from a source M to the test particle m located at the distance $\Delta x=\eta\lambda_m$ from the holographic screen of radius R covering M, is evaluated as [7,15]

$$F=T\frac{\Delta S}{\Delta x}. \tag{8}$$

Now, since $\frac{\Delta S}{\Delta x}=\frac{\Delta S}{\Delta A}\frac{\Delta A}{\Delta x}=(\frac{1}{\eta\lambda_m})\frac{dS}{dA}$ [7,15], using Eqs. (6) and (7), this equation can be written as

$$F=(\frac{1}{2\pi\eta})\frac{Mm}{R^2}\frac{dS}{dA}, \tag{9}$$

where $A=4\pi R^2$. We also used the Compton wavelength expression $\lambda_m=1/m$, as well as Eq. (7) to obtain Eq. (9).

3. Possible MOND theories

As we mentioned, the Rényi and Sharma-Mittal entropies can provide suitable description for the current accelerated universe and thus dark energy [53–56]. Here, using the introduced generalized entropy formalisms, Eq. (7), the $S_B=S_T$ relation [49], and Eq. (9), we are going to get the various MOND theories allowed by employing the Rényi and Sharma-Mittal entropies to the system.

3.1. Rényi entropy

Now, inserting Eq. (4) into equation (9), one can easily obtain

$$F=(\frac{1}{8\pi\eta})\frac{Mm}{R^2}\frac{1}{\delta A/4+1}. \tag{10}$$

Since we set $G=1$, we should have $F=\frac{Mm}{R^2}$ at the $\delta=0$ limit leading to $8\pi\eta=1$ in full agreement with [7,15]. Finally, defining $\mathcal{A}_0\equiv\delta\pi M$ and $a\equiv M/R^2$, one can rewrite Eq. (10) as

$$F=\left(\frac{1}{1+\mathcal{A}_0/a}\right)ma=f(a)ma, \tag{11}$$

which recovers the Newtonian gravity at the appropriate limit of $\delta\to 0$ (or equally $\mathcal{A}_0\to 0$). In addition, for $a\gg\mathcal{A}_0$ ($a\ll\mathcal{A}_0$), we have $F\simeq ma=Mm/R^2$ ($F\simeq m\frac{a^2}{\mathcal{A}_0}=Mm/\pi\delta R^4$) meaning that the Modified Newtonian force obtained in Eq. (11) has similarities with a MOND theory with simple interpolating function $\mu\left(\frac{\mathcal{A}_0}{a}\right)=1/(1+\mathcal{A}_0/a)$.

From the phenomenological point of view, following [59,60] and working with the accelerations, one can introduce vectors and divides Eq. (11) as

$$\vec{a}=\vec{a}_n+\vec{a}_{nn}, \tag{12}$$

where the total acceleration $\vec{a}$ is given by the ordinary Newtonian acceleration $\vec{a}_n$ plus the acceleration $\vec{a}_{nn}$ which is due to the non-Newtonian force. Now, taking the square of Eq. (12) one gets [59]:

$$\vec{a}_{nn}\cdot\vec{a}_n=\frac{1}{2}\left(a^2-a_n^2-a_{nn}^2\right). \tag{13}$$

In this equation the dot represents the three-dimensional scalar product. Equation (13) is a general relation which expresses the unknown vector $\vec{a}_n$ in terms of the total acceleration $\vec{a}$, of the acceleration due to the non-Newtonian force $\vec{a}_{nn}$ and of the magnitudes a^2, a_n^2 and a_{nn}^2. From Eq. (13), one obtains the acceleration $\vec{a}_n$ as being [59]

$$\vec{a}_n=\frac{1}{2}(a^2-a_n^2-a_{nn}^2)\frac{\vec{a}}{a_{nn}\cdot a}+\vec{b}\,\mathrm{x}\,\vec{a}_{nn}, \tag{14}$$

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