Physics Letters B ●●● (●●●●) ●●●–●●●

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Experimental constraints on the second clock effect

I.P. Lobo, C. Romero *

Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, CEP 58051-970, João Pessoa, PB, Brazil

A R T I C L E   I N F O

Article history:
Received 30 May 2018
Received in revised form 4 July 2018
Accepted 9 July 2018
Available online xxxx
Editor: M. Cvetič

Keywords:
Second clock effect
Weyl space–time
Unified field theories
Proper time
Gravitation
Muon decay

A B S T R A C T

We set observational constraints on the second clock effect, predicted by Weyl unified field theory, by investigating recent data on the dilated lifetime of muons accelerated by a magnetic field. These data were obtained in an experiment carried out in CERN aiming at measuring the anomalous magnetic moment of the muon. In our analysis we employ the definition of invariant proper time proposed by V. Perlick, which seems to be the appropriate notion to be worked out in the context of Weyl space–time.

1. Introduction

Since the advent of the special and general relativity the quest for the determination of the true geometric nature of space–time has long been a debated matter of research among theoretical physicists. The treatment of space–time as a differential manifold endowed with a Riemannian metric tensor, which obeys Einstein's field equations, still remains the paradigm of gravity theory. However, in recent years a great deal of effort has gone into the investigation of the so-called *modified gravity theories*, mainly motivated by attempts at explaining current data coming from observational cosmology as well as the important issues of dark matter and dark energy [1]. In this letter, however, we revisit some ideas developed by H. Weyl in his unified theory, one of the first modified gravity theories, which appeared soon after the birth of general relativity [2]. Weyl's theory encountered a severe objection put forward by Einstein, who believed that it would lead to a physical effect not yet observed (the so-called *second clock effect*). Curiously, as far as we know, neither theoretical calculations nor any experimental attempt at measuring the magnitude of the predicted effect has been carried out up to now.

Let us now briefly recall some basic tenets of the geometry conceived by H. Weyl which underlies his unified theory. Perhaps the main feature of this geometry is the fact that a vector can have its length changed when parallel transported along a curve, which is a consequence of the presence of a 1-form field in the compatibility condition between the metric and the affine connection. The existence of a group of transformations that leaves this new compatibility condition invariant is another interesting fact noticed by Weyl, which ultimately led to the discovery of the gauge theories [3]. As is well known, Weyl's idea was to give a geometric character to the electromagnetic potential by identifying it with a purely geometric 1-form field. He then proposed an invariant action that contained both the gravitational and the electromagnetic fields. However, Einstein pointed out that the non-integrability of length, a characteristic of Weyl space–time, would imply that the rate at which a clock measures time, i.e. its clock rate, would depend on the past history of the clock. As a consequence, spectral lines with sharp frequencies would not appear [2]. This came to be known in the literature as the *second clock effect* [4]. (The *first clock effect* refers to the well-known effect corresponding to the "twin paradox" predicted by special and general relativity theories.)

Despite the fact that this essentially qualitative objection has led to a rejection of Weyl theory as being non-physical, an actual measurement of the magnitude of the second-clock effect predicted by Weyl theory has never been carried out. Moreover, worse than that, as far as we know even the concept of proper time measured by an ideal clock in Weyl theory has never been discussed, neither by Einstein nor by Weyl himself. In fact, the usual definition of proper time adopted in general relativity as the arc-length of a curve (the clock hypothesis) cannot be properly carried

* Corresponding author.
E-mail addresses: iarley_lobo@fisica.ufpb.br (I.P. Lobo), cromero@fisica.ufpb.br (C. Romero).

https://doi.org/10.1016/j.physletb.2018.07.019

over to Weyl geometry for the simple reason that this definition is not invariant under Weyl transformations (see [5,6] and references therein). It turns out, however, that this problem has been finally settled by V. Perlick, who proposed a definition of proper time which is consistent with Weyl's principle of invariance [7,8]. Perlick's notion of proper time provides a correction to the arc-length formula, and reduces to the general relativistic proper time when the Weyl 1-form field vanishes. Moreover, it can be used to set experimental bounds on the predicted second clock effect. Following a renewed interest in Weyl theory, we believe that attempts to detect the possible existence of the second clock effect is of interest in its own, and may lead to results of physical relevance whose significance may lie beyond any particular gravity theory.

In this letter, we propose to use as our standard clocks unstable particles by investigating the effect of an external magnetic field on their dilated lifetime. Specifically, our aim is to set an experimental constraint on the second clock effect by looking at the Perlick's proper time corresponding to the dilated lifetime of muons accelerated by this magnetic field.

2. Weyl geometry

As we have mentioned before, the basic idea of Weyl geometry is the introduction of a 1-form field σ_α (called the Weyl field), which is used to replace the Riemannian compatibility condition between the metric $g_{\mu\nu}$ and the connection ∇_α by requiring that the new condition reads

$$\nabla_\alpha g_{\mu\nu} = \sigma_\alpha g_{\mu\nu}. \tag{1}$$

Weyl then found out that by performing the simultaneous transformations

$$\bar{g}_{\mu\nu} = e^f g_{\mu\nu}, \tag{2a}$$

$$\bar{\sigma}_\alpha = \sigma_\alpha + \partial_\mu f, \tag{3}$$

where $f = f(x)$ is an arbitrary scalar function, the compatibility condition (1) is preserved, i.e., we have $\nabla_\alpha \bar{g}_{\mu\nu} = \bar{\sigma}_\alpha \bar{g}_{\mu\nu}$. The discovery of this invariance is generally considered to be the birth of modern gauge theories (see [3] and references therein). It turns out then that the condition (1) leads to a new kind of curvature, given by $F_{\mu\nu} = \partial_\mu \sigma_\nu - \partial_\nu \sigma_\mu$, called by Weyl *the length curvature*, which is invariant under (3). These findings led Weyl to identify the 1-form σ_α with the 4-potential A_α of the electromagnetic field [2] by writing

$$\sigma_\alpha = \lambda A_\alpha, \tag{4}$$

where the constant λ is introduced just for dimensional reasons since σ_α has dimensions of $[\text{length}]^{-1}$ (of course, it is always possible to choose units such that $\lambda = 1$).

The length curvature can be viewed as a measure of the non-integrability of vector lengths when a vector field is parallel transported around a loop. For instance, let V^μ be the components with respect to a coordinate basis of a time-like vector V that is parallel transported around a closed curve $\gamma \mapsto \gamma(t) : \gamma[a,b] \in R \to M$ (with $\gamma(a) = \gamma(b)$). If we denote $L^2 = g_{\mu\nu} V^\mu V^\nu$ then it can easily be shown that

$$L(a) = L(b) \exp\left[\frac{1}{2} \oint \sigma_\mu \frac{dx^\mu}{dt} dt\right], \tag{5}$$

where $d\gamma/dt \doteq (dx^\mu/dt)\partial_\mu$, $L(a)$ and $L(b)$ denote the initial and final length of V, respectively. Surely, $L(a) = L(b)$ if and only if there exists a scalar function ϕ such that $\sigma_\mu = \partial_\mu \phi$. Clearly, in this case, from Stokes' theorem, $F_{\mu\nu} = \partial_\mu \sigma_\nu - \partial_\nu \sigma_\mu$ must vanish and we end up with a *Weyl Integrable space–time (WIST)*.

We could say that the non-integrability of lengths is in the root of the already mentioned Einstein's objection to Weyl's theory. Indeed, Einstein argued that this predicted effect implies that the clock rate of atomic clocks should be path dependent. In fact, Einstein's reasoning is based on two hypotheses:

a) The proper time $\Delta\tau$ measured by a clock traveling along a curve $\gamma = \gamma(t)$ is given as in general relativity, that is, by the (Riemannian) prescription

$$\Delta\tau = \frac{1}{c} \int [g(V,V)]^{\frac{1}{2}} dt = \frac{1}{c} \int \left[g_{\mu\nu} V^\mu V^\nu\right]^{\frac{1}{2}} dt, \tag{6}$$

where V denotes the vector tangent to the clock's world line and c is the speed of light. This assumption is known as the *clock hypothesis* and assumes that the proper time only depends on the instantaneous speed of the clock and on the metric field.

b) The fundamental clock rate of standard clocks is given by the (Riemannian) length $L = \sqrt{g(V,V)}$ of a certain vector V.

However, it has been argued recently that in order to discuss the existence of the second clock effect a new notion of proper time, consistent with *Weyl's Principle of Gauge Invariance*,[1] is needed [5]. It happens to be that such a notion exists and was recently given by V. Perlick [7].

Let us now briefly recall the notion of proper time proposed by V. Perlick. First, let us define a *standard clock* according to the following definition: A time-like curve $\gamma : \gamma[a,b] \in R \to M$, $t \mapsto \gamma(t)$, is called a *standard clock* if $\frac{D\gamma'}{dt}$ is orthogonal to $\gamma'(t)$, i.e. $g(\gamma', \frac{D\gamma'}{dt}) = 0$. We will then say that a time-like curve γ is parametrized by proper time if the parametrized curve is a standard clock. It can be shown that from this definition it follows that the proper time elapsed between two events corresponding to the parameter values t_0 and t in the curve γ is given by

$$\Delta\tau(t) = \left(\frac{d\tau/dt}{\sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta}}\right)_{t=t_0} \int_{t_0}^{t} \exp\left(-\frac{1}{2}\int_{u_0}^{u} \sigma_\rho \dot{x}^\rho ds\right) \left[g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right]^{1/2} du, \tag{7}$$

where the overdot means derivative with respect to the curve's parameter [8]. It has also been shown that Perlick's time has all the properties a good definition of proper time in a Weyl space–time should have, such as, Weyl-invariance, positive definiteness, additivity. In addition to that, in the limit in which the length curvature $F_{\mu\nu}$ goes to zero Perlick's time reduces to the Riemannian or WIST proper time. Recently, it was shown the equivalence between this definition and the one given in the well-known paper by Ehlers, Pirani, and Schild (EPS) [8,9]. The latter was entirely based on axiomatic approach which leads to a Weyl structure as the most suitable model for space–time.

Another important property of Perlick's hypothesis (perhaps unexpected) concerning the proper time of a standard clock is that it also predicts the existence of the second clock effect, namely, that the clock rate of a local observer depends on its path [8]. More precisely, consider two clocks c_1 and c_2 synchronized at point A (see Fig. 1), which are transported together until point B, then separated and transported along two different paths, Γ_1 and Γ_2, until

[1] The *Principle of Gauge Invariance* asserts that all physical quantities must be invariant under the gauge transformations. This principle was strictly followed by Weyl and guided him when he had to choose an action for his theory. It should also be noted here that any invariant scalar of this geometry must necessarily be formed by both the metric $g_{\mu\nu}$ and the Weyl gauge field σ_μ.

Please cite this article in press as: I.P. Lobo, C. Romero, Experimental constraints on the second clock effect, Phys. Lett. B (2018), https://doi.org/10.1016/j.physletb.2018.07.019