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Pion–nucleon sigma term revisited in covariant baryon chiral perturbation theory

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ABSTRACT

We study the latest $N_f = 2+1+1$ and $N_f = 2$ ETMC lattice QCD simulations of the nucleon masses and extract the pion–nucleon sigma term utilizing the Feynman–Hellmann theorem in SU(2) baryon chiral perturbation theory with the extended-on-mass-shell scheme. We find that the lattice QCD data can be described quite well already at the next-to-next-to-leading order. The overall picture remains essentially the same at the next-to-next-to-next-to-leading order. Our final result is $\sigma_{\pi N} = 50.2(1.2)(2.0)$ MeV, or equivalently, $f^N_{u/d} = 0.0535(13)(21)$, where the first uncertainty is statistical and second is theoretical originated from chiral truncations, which is in agreement with that determined previously from the $N_f = 2+1$ and $N_f = 2$ lattice QCD data and that determined by the Cheng–Dashen theorem. In addition, we show that the inclusion of the virtual $\Delta(1232)$ does not change qualitatively our results.

1. Introduction

In recent years, the pion–nucleon sigma term has attracted much attention, partly because of its role in predicting the cross section of certain candidate dark matter particles interacting with the nucleons [1]. Historically, a "canonical value" of the pion–nucleon sigma term $\sigma_{\pi N} = m_l \langle N|\bar{u}u + \bar{d}d|N\rangle \sim 45$ MeV was derived in Ref. [2] from the pion–nucleon scattering data. Later, an updated analysis of πN scattering yielded a larger value $\sigma_{\pi N} = 64(8)$ MeV [3]. In the past few years, several phenomenological studies of pion–nucleon scattering using chiral perturbation theory (ChPT) and/or Roy–Steiner equations, e.g. Refs. [4–8], have derived a $\sigma_{\pi N}$ around 60 MeV. In the meantime, the pion–nucleon sigma term has also been extensively studied in lattice quantum chromodynamics (lattice QCD) by either computing three-point (the direct method) [9–13] or two-point correlation functions (the so-called spectrum method) [14–25]. Due to the many systematic and statistical uncertainties inherent in these studies, no consensus has been reached on the precise value of the pion–nucleon sigma term, although several recent studies seem to prefer a small value~ 40 MeV [11–13,25]. Apparently, there exists a tension between the pion–nucleon sigma term determined from the phenomenological studies and that from the lattice QCD simulations.

As stressed in Ref. [23], two key factors are important in a reliable and accurate determination of the pion–nucleon sigma term using the lattice nucleon mass data with the spectrum method, i.e., lattice QCD simulations with various setups and configurations and a proper formulation to parameterize the pion-mass dependence of the nucleon mass. For the later, baryon chiral perturbation theory (BChPT), an effective field theory of low-energy QCD, provides a model-independent framework to study the pion-mass dependence of the nucleon mass. In the last few years, the European Twisted Mass Collaboration (ETMC) has performed several lattice QCD studies to extract the nucleon mass with the $N_f = 2$ [26,27] and $N_f = 2+1+1$ [24] twisted mass fermions. Since the dynamical strange and charm quarks have minor impact on the ETMC nucleon masses, in a recent work, Alexandrou et al. (ETMC) performed a combined fit to the 17 sets of the $N_f = 2+1+1$ nucleon masses and one $N_f = 2$ physical ensemble using SU(2) BChPT,[1] and predicted a pion–nucleon sigma term 64.9(1.5)(13.2) MeV [27]. This value is much larger than that obtained from the

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[1] In principle, the twisted-mass ChPT [28,29] is more suitable for the analysis of the ETMC data.

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direct method with the ensemble at the physical point by the same collaboration, $\sigma_{\pi N} = 37.2(2.6)^{(4.7)}_{(2.9)}$ [12]. However, ones should note that the large $\sigma_{\pi N}$ of Ref. [27] was obtained in the spectrum method using the heavy baryon (HB) chiral perturbation theory, which is known to perform sometimes badly in terms of convergence (see, e.g., Ref. [30,31]). Particularly, it was shown in Ref. [27] that at next-to-next-to-leading order (NNLO) the best fit yields a χ^2/d.o.f. ≈ 1.6 while only at "next-to-next-to-next-to leading order (N^3LO)",[2] a χ^2/d.o.f. ≈ 1.1 can be achieved.

Since the determination of the pion–nucleon sigma term via the Feynman–Hellmann theorem is sensitive to the extracted pion-mass dependence of the nucleon mass from the lattice QCD data, a better description of the ETMC data is needed. Therefore, it is timely and worthy to reanalyze the same lattice QCD data as Ref. [27] using covariant baryon chiral perturbation theory with the extended-on-mass-shell (EOMS) scheme [32], which has shown a number of both formal and practical advantages and has solved a number of long-existing puzzles in the one-baryon sector [33]. Furthermore, the applications of the EOMS BChPT in the studies of the lattice QCD octet baryon masses turn out to be very successful as well [20,21,34,35].[3] Therefore, in this work, we employ the two-flavor covariant BChPT to calculate the nucleon mass up to N^3LO. It is shown that we can achieve a better description of the 18 sets of ETMC data, i.e. χ^2/d.o.f. ≤ 1.0, in comparison with the study in the HB scheme [27]. With the obtained LECs, we predict a pion–nucleon sigma term, $\sigma_{\pi N} = 50.2(1.2)(2.2)$ MeV, using the Feynman–Hellmann theorem.

This paper is organized as follows. In Section 2, we briefly summarize the theoretical ingredients needed to analyze the ETMC lattice QCD data. In Section 3, we perform fits to them following the strategy of Ref. [27] and predict the pion–nucleon sigma term using the Feynman–Hellmann theorem. The so-obtained low-energy constants (LECs) are then used to calculate the scattering length as well as the pion–nucleon sigma term with the Cheng–Dashen theorem. In Section 4, a short summary is given.

2. Theoretical framework

The nucleon mass has been calculated up to $\mathcal{O}(p^4)$ both in the two-flavor sector [21] and in the three-flavor sector [20] in covariant BChPT with the EOMS scheme. To make the present work self-consistent, we spell out the nucleon mass up to $\mathcal{O}(p^4)$, which in the isospin symmetric limit reads

$$m_N = m_0 - 4c_1 m_\pi^2 + \alpha m_\pi^4 + \frac{3c_2 m_\pi^4}{128\pi^2 f_\pi^2} - \frac{3}{64\pi^2 f_\pi^2}(8c_1 - c_2 - 4c_3) m_\pi^4 \left(1 + \log\frac{\mu^2}{m_\pi^2}\right) + \frac{3g_A^2}{4(4\pi f_\pi)^2}\left[H_N^{(3)}(m_0,\, m_\pi,\, \mu) + H_N^{(4)}(m_0,\, (-4c_1 m_\pi^2),\, m_\pi,\, \mu)\right], \quad (1)$$

where f_π is the pion decay constant in the chiral limit, and g_A is the axial coupling. There are four LECs, c_1, c_2, c_3, and α. The two loop functions, $H_N^{(3)}$ and $H_N^{(4)}$, are the contributions of the $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ one-loop diagrams with the power-counting breaking terms subtracted [20,21]

$$H_N^{(3)} = -\frac{2m_\pi^3}{m_0}\left[\frac{m_\pi}{2}\log\frac{m_\pi^2}{m_0^2} + \sqrt{4m_0^2 - m_\pi^2}\times \left(\arctan\frac{m_\pi}{\sqrt{4m_0^2 - m_\pi^2}} - \arctan\frac{m_\pi^2 - 2m_0^2}{m_\pi\sqrt{4m_0^2 - m_\pi^2}}\right)\right], \quad (2)$$

$$H_N^{(4)} = \frac{2m_\pi^3}{m_0^2\sqrt{4m_0^2 - m_\pi^2}}(4c_1 m_\pi^4)\arccos\frac{m_\pi}{2m_0} - m_\pi^2\left[\frac{4c_1 m_\pi^4}{m_0^2}\log\frac{m_\pi^2}{m_0^2} - 8c_1 m_\pi^2 \log\frac{m_0^2}{\mu^2}\right], \quad (3)$$

which are calculated in the dimensional regularization scheme with the renormalization scale μ. Following Ref. [40], we take $f_\pi = 0.0871$ GeV, $g_A = 1.267$, and $\mu = 1.0$ GeV in our numerical study, unless otherwise specified.

In principle, the four LECs (c_i and α) can be calculated directly from QCD. However, because of the nonperturbative nature of QCD at low energies, one usually determines their value by performing a least-square fit to the lattice QCD nucleon masses and/or experimental data. It was shown in Refs. [20,22] that finite volume corrections need to be taken into account, particularly for the $m_\pi L < 4$ ensembles, in order to describe the lattice QCD data with a $\chi^2 \approx 1.0$. In the present case, since some of the ETMC results are obtained with $m_\pi L < 4$, we take the finite volume corrections into account up to $\mathcal{O}(p^4)$, which read

$$\delta m_N = \frac{3g_A^2}{4f_\pi^2}\left(\delta H_N^{(3)} + \delta H_N^{(4)}\right) + \frac{3}{2f_\pi^2}\left[2c_1 m_\pi^2 \delta_{1/2}(m_\pi^2) - c_2 \delta_{-1/2}(m_\pi^2) - c_3 m_\pi^2 \delta_{1/2}(m_\pi^2)\right], \quad (4)$$

with

$$\delta_r(\mathcal{M}^2) = \frac{2^{-1/2-r}(\sqrt{\mathcal{M}^2})^{3-2r}}{\pi^{3/2}\Gamma(r)}\times \sum_{\vec{n}\neq 0}(L\sqrt{\mathcal{M}^2}|\vec{n}|)^{-3/2+r} K_{3/2-r}(L\sqrt{\mathcal{M}^2}|\vec{n}|), \quad (5)$$

where $K_n(z)$ is the modified Bessel function of the second kind, and

$$\sum_{\vec{n}\neq 0} \equiv \sum_{n_x=-\infty}^{\infty}\sum_{n_y=-\infty}^{\infty}\sum_{n_z=-\infty}^{\infty}(1 - \delta(|\vec{n}|, 0)), \quad (6)$$

with $\vec{n} = (n_x, n_y, n_z)$. The finite volume correction of the one-loop diagrams, $\delta H_N^{(3)}$ and $\delta H_N^{(4)}$, are calculated in Refs. [20,42] and read

$$\delta H_N^{(3)} = -\int_0^1 dx\left[\frac{1}{2}m_0(2x+1)\delta_{1/2}(\mathcal{M}_N^2) - \frac{1}{4}m_0\left(m_0^2 x^3 + \mathcal{M}_N^2(x+2)\right)\delta_{3/2}(\mathcal{M}_N^2)\right], \quad (7)$$

and

[2] One should note that this is not a complete N^3LO study in HB ChPT, since the contributions from the $\mathcal{O}(p^4)$ tadpole and mass-insertion loop diagrams were not included.

[3] It has been extended to heavy flavor sectors in recent years, see, e.g., Refs. [36–39].