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A new method to study the number of colors in the final-state interactions of hadrons

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ABSTRACT

Based on Chiral Perturbation Theory we introduce the dependence on the number of colors (N_C) for the $\pi\pi \to \pi\pi$ scattering amplitudes. Those amplitudes are calculated from dispersion relations that respect analyticity and coupled channel unitarity, as well as accurately describing experiment. By varying N_C the trajectories of the poles and residues (the couplings to $\pi\pi$) of the light mesons, the $\sigma/f_0(500)$, $f_0(980)$, $\rho(770)$ and $f_2(1270)$ are investigated. Our results show that the method proposed is a reliable way to study the N_C dependence in hadron–hadron scattering with final-state interactions.

The lightest scalar mesons are rather interesting as they have the same quantum numbers as the QCD vacuum. The nature of them is still a mystery [1–4]. The phenomenology of these is complicated due the contribution from important final-state interactions (FSI) [5,6]. Dispersion relations are the natural way to include FSI, see e.g. [7,8]. For some of the light mesons, like the σ, κ, their existence has been confirmed [9–12] and accurate pole locations and $\pi\pi$ couplings, including also the $\rho(770)$, have been given in Refs. [13–15]. Concerning the nature of the scalar mesons, there are a cornucopia of models [16–35]. Among them tracking the large N_C trajectories of the poles is an effective diagnostics to distinguish ordinary from non-ordinary quark–antiquark structure as considered in [36–40]. However, these analyses based on unitarized Chiral Perturbation theory (UχPT) lack crossing symmetry. Unitarization itself will also generate spurious poles [41] and cuts [42,40]. In contrast, dispersion relations respect analyticity, but including coupled channel unitarity and the N_C dependence is difficult. Clearly, both analyticity and coupled channel unitarity are critical in the region of the $\bar{K}K$ threshold, close to which the $f_0(980)$ is located. To solve this problem, we use an Omnès representation based on the phase of the relevant amplitudes, rather than the elastic phase shift [43,44]. There has been renewed interest in the study of the large N_C limit [45,46] of the properties of resonances [47–49]. Weinberg [50] pointed out that resonant tetraquark states could exist due to the contribution of the leading order (LO) 'connected' diagrams to the Green functions. Their widths are $\mathcal{O}(N_C^{-1})$, as narrow as ordinary mesons. They could be even narrower, with width of $\mathcal{O}(N_C^{-2})$, when the flavor of the quarks is combined in different ways [51]. The width could also be wide as $\mathcal{O}(1)$, see [52]. There are many other interesting discussions such as [53,54] and references therein. In this paper we focus on establishing a 'practical' way to study the N_C dependence of the scattering amplitudes, built into dispersion relations. Resonances appearing in the intermediate states are also studied.

In this letter we first use dispersive methods to obtain the $\pi\pi$ scattering amplitudes up to 2 GeV. We construct the amplitudes in a model-independent way, which is both analytic and respects coupled channel unitarity. We also recalculate the analytical expressions of the $IJ = 00, 02, 11$ waves in $SU(3)$ Chiral Perturbation Theory (χPT) up to $\mathcal{O}(p^4)$. By matching with the χPT amplitudes up to $\mathcal{O}(N_C^{-1})$, we introduce the N_C-dependence into the dispersive amplitudes. This N_C dependence is automatically transferred to the high-energy region, where the FSI are implemented by a dispersion relation. We give the trajectories of the poles and residues by varying N_C. The behavior of the $\rho(770)$, $f_2(1270)$, $\sigma/f_0(500)$ and $f_0(980)$ show that this is a reliable way to study the number of colors in hadron-hadron scattering. The N_C trajectory of the light scalar mesons supports a mixed structure of hadronic molecule and $\bar{q}q$ components (for a recent review on hadronic

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Table 1
The fit parameters as given in Eq. (3). The errors are given by MINUIT and notice that the $\alpha_{1,2}$ are fixed by the scattering lengths and slope parameters [44,57,58].

	${T_{11}}_S^0$	${T_{11}}_D^0$	${T_{11}}_P^1$
α_1	2.4051	0.2972	0.4283
α_2	−1.9451	−0.9354	−0.2976
α_3	1.3473(36)	2.1931(2)	0.6173(16)
α_4	−0.4629(19)	−2.6108(2)	−0.7092(11)
α_5	0.0038(6)	1.6508(1)	0.3774(4)
α_6	0.0307(2)	−0.5679(1)	−0.0909(1)
α_7	−0.0045(1)	0.1004(1)	0.0081(1)
α_8	–	−0.0071(1)	–

molecules, see Ref. [55]), while a tetra-quark component is also possible.

We first present our $IJ = 00, 02, 11$ partial waves of $\pi\pi \to \pi\pi$ calculated in a model-independent way. We start from:

$$T^I_J(s) = P^I_J(s)\Omega^I_J(s), \tag{1}$$

where $\Omega^I_J(s)$ is the Omnès function [56]:

$$\Omega^I_J(s) = \exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\varphi^I_J(s')}{s'(s'-s)}\right), \tag{2}$$

with $\varphi^I_J(s)$ the phase of the partial wave amplitude $T^I_J(s)$, which has been given in previous amplitude analysis [43,44]. This phase is known from experiment up to roughly 2 GeV, while in the higher energy region it is constrained by unitarity. The Omnès function is truncated at $s = 22$ GeV^2. The function $P^I_J(s)$ includes the effect of the left-hand cut (l.h.c) and corrections that come from the distant right-hand cut (r.h.c) above 2 GeV. The latter one has a tiny contribution to the region where the light mesons appear. Other information is provided by chiral dynamics that fixes the Adler zero in the S-wave, and the approach to the threshold of the S-, P-, and D-waves in terms of scattering lengths and effective ranges. We therefore parameterize the $P^I_J(s)$ as

$$P^I_J(s) = (s - z^I_J)^{n_J} \sum_{k=1}^{n} {\alpha^I_J}_k (s - 4M_\pi^2)^{k-1}, \tag{3}$$

where z^I_J is the Adler zero for the S-wave and $4M_\pi^2$ for P- and D-waves. The parameter n_J is 1 for S- and P-waves and 2 for D-waves. The parameters α_i are given in Table 1. The units of the α_k are chosen to ensure that the amplitude $T^I_J(s)$ is dimensionless.

The fit results are shown in Fig. 1. We fit the amplitudes in the region of $s \in [0, 4$ GeV$^2]$, where the 'data' is as follows: χPT amplitudes at $[0, 4M_\pi^2]$, amplitudes of K-matrix and Roy-like equation at $[4M_\pi^2, 2$ GeV$^2]$, and experiment data up to 4 GeV^2. The fits are of high quality, even in our 'prediction' region where $s \in [-4M_\pi^2, 0]$. In this region the real part of our amplitudes is in good agreement with that of χPT ($\mathcal{O}(p^4)$), and the imaginary part certainly vanishes. Notice that the imaginary part of the χPT amplitudes is rather small, too.

There is no l.h.c in our parametrization,[1] but its contribution to the shaded region on the complex s-plane, as shown in Fig. 2, are properly implemented, as we fit our amplitudes to the data as well as the amplitudes of Roy equation [10] in the presence of crossing symmetry.[2] We note that the amplitudes on the upper half of s-plane are readily obtainable from the ones on the lower side according to the Schwarz reflection principle. From Fig. 2 we see that our amplitudes are compatible to that of the Roy equation analysis in the complex s-plane. The distribution of contours is in good agreement and moreover, their gradient variations are compatible with each other, as shown by the shading of the color from blue to red. Nevertheless, amplitudes on the edge of the domain and in the region of Im$s < -0.3$ GeV^2 for $T^0_S(s)$ are less consistent with differences ≤ 0.1. The difference around $\sqrt{s} \simeq 1$ GeV of $T^0_S(s)$ are also not ignorable, this is caused by the different treatment of the $K\overline{K}$: ours include the physical K^+K^- and $K^0\overline{K}^0$ [43], while it is treated as $K\overline{K}$ in the isospin basis in the analysis of the Roy equations [10].

Table 2
The pole locations and residues on the second Riemann sheet.

State	Pole locations (MeV)	$g_{f\pi\pi} = \lvert g_{f\pi\pi}\rvert e^{i\varphi}$	
		$\lvert g_{f\pi\pi}\rvert$ (GeV)	φ (°)
$\sigma/f_0(500)$	$436.2(12.2) - i260.7(6.8)$	0.45(0.02)	74(2)
$f_0(980)$	$997.7(1.1) - i21.7(1.9)$	0.27(0.02)	84(3)
$f_0(1370)$	$1431.6(34.6) - i185.4(22.4)$	0.78(0.21)	−47(18)
$f_2(1270)$	$1278.3(7.0) - i79.3(17.8)$	0.50(0.05)	0.7(4.3)
$\rho(770)$	$762.4(3.9) - i68.7(6.3)$	0.34(0.01)	12(3)

With these amplitudes, we can extract the poles and residues on the second sheet. The residue $g_{f\pi\pi}$ and pole s_R on the second Riemann sheet are defined as

$$T^{II}(s) = \frac{g^2_{f\pi\pi}}{s_R - s}. \tag{4}$$

The pole locations and their residues are listed in Table 2. These are very similar from those of previous analyses [13,15,43,64,65]. For the $f_0(1370)$ and $f_2(1270)$, to find the pole closest to the physical sheet one needs to include the $\pi\pi$, $\overline{K}K$, 4π as coupled channels. The pole obtained here from a single $\pi\pi$ channel is of course, not on the Riemann sheet which is closest to the physical region. Therefore, we do not discuss the $f_0(1370)$ in the next sections. For the $f_2(1270)$, $\pi\pi$ is the dominant decay channel and the pole on the second sheet is not far away from the physical one.

Having analytically calculated the partial wave amplitudes of $IJ = 00, 11, 02$ within one-loop SU(3) χPT, we can match our dispersive results to these and so fix their N_C dependence. We note several points about this matching: First, though the matching is done in the low-energy region, this N_C dependence is transferred to high-energy region. As the FSI of hadrons in the higher energy region correspond to 'hadron loop' corrections, which could be translated into higher order corrections (quark loops) in large N_C QCD, and are thus suppressed by an extra factor of N_C^{-1}. Of course, this N_C dependence is energy-dependent, see the discussion below. Second, we only calculate the χPT amplitudes up to $\mathcal{O}(p^4)$. Here, the imaginary part of the χPT amplitudes is given entirely from one-loop integrals and thus both l.h.c and r.h.c are N_C^{-2}, while the real part is N_C^{-1}. For $\mathcal{O}(p^6)$, it is still N_C^{-1} for the real part of the amplitudes as given by the contact terms, and again it is N_C^{-2} for the imaginary part. The latter one is either from the 2-loop corrections or from the 1-loop corrections with one insertion of a contact term of $\mathcal{O}(p^4)$. This N_C dependence continues when one goes to higher orders. One thus finds that $\mathrm{ImT}/\mathrm{ReT} \sim 1/\mathrm{N_C}$ and we define:

[1] It is worth to note that in [13] the mass and width of σ deviates about 15% from the original value when the l.h.c is removed.

[2] The comparison of the I=1 P-wave amplitudes has already been given in [59]. However, we present it here for convenience. Notice that the D-wave is absent in the Roy equation analysis[13], and the $f_2(1270)$ is quite far away from the l.h.c. We thus do not discuss it here.