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Renormalon free part of an ultrasoft correction to the static QCD potential

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ABSTRACT

Perturbative calculations of the static QCD potential have the $u = 3/2$ renormalon uncertainty. In the multipole expansion performed within pNRQCD, this uncertainty at LO is known to get canceled against the ultrasoft correction at NLO. To investigate the net contribution remaining after this renormalon cancellation, we propose a formulation to separate the ultrasoft correction into renormalon uncertainties and a renormalon independent part. We focus on very short distances $\Lambda_{\rm QCD} r \lesssim 0.1$ and investigate the ultrasoft correction based on its perturbative evaluation in the large-β_0 approximation. We also propose a method to examine the local gluon condensate, which appears as the first nonperturbative effect to the static QCD potential, without suffering from the $u = 2$ renormalon.

1. Introduction

The static QCD potential plays an important role to investigate the QCD dynamics. It has been investigated extensively by using perturbation theory, effective field theory and lattice simulations.

In perturbative evaluations, perturbative coefficients are expected to show factorial behaviors at large orders. Such divergent behaviors, in particular those related to a positive renormalon, induce ambiguity to the resummation of the perturbative series [1]. For the static QCD potential, renormalons are located at positive half integers in the Borel u-plane. The first renormalon at $u = 1/2$ causes an uncertainty to the r-independent constant of the potential. This renormalon is known to get canceled in the total energy (i.e. the sum of the QCD potential and twice the pole mass) within usual perturbation theory once the pole mass is expressed as a perturbative series in terms of the $\overline{\rm MS}$ mass.

In considering cancellation of the other renormalons, it is useful to adopt the effective field theory (EFT) known as potential nonrelativistic QCD (pNRQCD) [2]. The renormalons are expected to get canceled ultimately in the multipole expansion performed in this EFT. The leading order (LO) term of this expansion is the singlet potential $V_S(r)$, which behaves as $\mathcal{O}(1/r)$ and can be evaluated in perturbation theory. The renormalons for V_S are located at positive half integers as mentioned above, and the leading r-dependent uncertainty is caused by the $u = 3/2$ renormalon. The next-to-leading order (NLO) term in the multipole expansion, which we denote by $\delta E_{\rm US}$, represents the dynamics at the ultrasoft scale $\sim \alpha_s(r^{-1})/r$ and its explicit r-dependence is $\mathcal{O}(r^2)$. In Ref. [2], it was pointed out within the leading-logarithmic approximation that the $u = 3/2$ renormalon exists in $\delta E_{\rm US}$ and it cancels against the $u = 3/2$ renormalon of V_S. In Ref. [3], the perturbative evaluation of $\delta E_{\rm US}$ in the large-β_0 approximation was completed, and again, the $u = 3/2$ renormalon cancellation was confirmed.

Although the $u = 3/2$ renormalon cancellation has been established, it has not been clarified what remains in the NLO calculation, $V_S + \delta E_{\rm US}$, as a consequence of this renormalon cancellation. In particular, the net contribution to $\delta E_{\rm US}$ remaining after this cancellation has not been made clear. In this Letter, we investigate the net contribution to $\delta E_{\rm US}$, which is not affected by the renormalon cancellation. The distances considered here are $\Lambda_{\rm QCD} r \lesssim 0.1$, where $\delta E_{\rm US}$ as well as V_S can be evaluated perturbatively since the ultrasoft scale satisfies $\alpha_s(r^{-1})/r \gg \Lambda_{\rm QCD}$. In this range, the leading nonperturbative correction is given through the local gluon condensate.[1] In order to examine the local gluon condensate, the perturbative part, i.e. $V_S + \delta E_{\rm US}$, should be clearly known in advance. Although the currently available order of perturbative expansion is far from the (expected) relevant order to the $u = 3/2$ renor-

[1] The appearance of this nonperturbative effect has been first considered in Refs. [4–6], and can be understood in a systematic expansion of pNRQCD [2].

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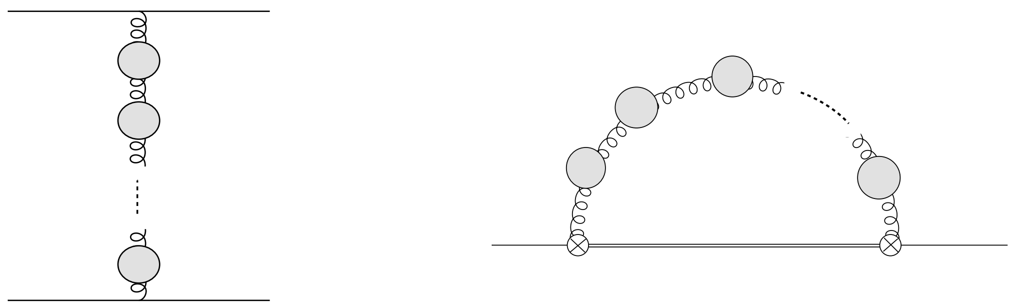

Fig. 1. Diagrams for the singlet potential (left) and δE_{US} (right) in the large-β_0 approximation.

malon,[2] this Letter aims at promoting theoretical understanding of the static QCD potential without suffering from renormalons.

To investigate the net NLO correction, we propose a formulation to separate δE_{US} into its renormalon uncertainties and a renormalon free part. In perturbative evaluations, the large-β_0 approximation is used. For the LO term V_S, a renormalon separation has been performed in Ref. [7]. In this Letter, we will see that the $u = 3/2$ renormalon uncertainty of V_S separated out in Ref. [7] is canceled against that of δE_{US} identified here. As a result, a theoretical expression after the $u = 3/2$ renormalon cancellation is obtained by the sum of the renormalon free parts of V_S and δE_{US}.[3] Each is presented in an analytic form in this Letter.[4]

Once the renormalon separation of δE_{US} is performed, it is straightforward to cope with the residual renormalon at $u = 2$. This renormalon in δE_{US} (found explicitly in Ref. [3]) is consistent with the fact that the local gluon condensate appears as the first nonperturbative effect. The $u = 2$ renormalon induces an $\mathcal{O}(\Lambda_{\mathrm{QCD}}^4 r^3)$ error to $V_S + \delta E_{\mathrm{US}}$, which is the same magnitude as the term of the local gluon condensate. Hence, the $u = 2$ renormalon is an obstacle in examining the local gluon condensate even after the $u = 3/2$ renormalon is removed. We circumvent this problem by including the $u = 2$ renormalon uncertainty of δE_{US} in the local gluon condensate, which results in the cancellation of the $u = 2$ renormalon in the local gluon condensate. This renormalon cancellation is explicitly confirmed in this Letter using the large-β_0 approximation. As a result, one can obtain the expansion in r up to the order including the local gluon condensate such that each term does not have the $u = 3/2$ and $u = 2$ renormalons. Such a result can be used to extract the local gluon condensate numerically, for instance, by comparing lattice simulations with the calculation presented here.

Our formulation to extract a renormalon free part of δE_{US} is an extension of Refs. [8,9], which propose the method to extract a renormalon free part in the leading term in operator product expansion (see Ref. [10] as a related work). The characteristics of our formulation is to introduce explicit cutoff scales, which are compatible with the concept of the EFT.[5] This clarifies intuitively how renormalon uncertainties appear and also vanish when combined with contributions of different energy scales. In particular, we will see how a renormalon free part is identified in connection with the cutoff scales.

[2] For the singlet potential V_S, the relevant perturbation order to the $u = 3/2$ renormalon is roughly estimated as $n_* = \frac{6\pi}{\beta_0 \alpha_s(r^{-1})} \sim 20$, while the exact series is currently known up to $\mathcal{O}(\alpha_s^4)$.

[3] The $u = 1/2$ renormalon uncertainty in V_S is just omitted as it changes only the r-independent constant.

[4] The analytical result for V_S that is free from renormalons has been given in Ref. [7].

[5] The cutoff scale dependence vanishes in the final results.

2. Extraction of renormalon free part

In the multipole expansion performed within pNRQCD, the static QCD potential is represented as [2]

$$V_{\mathrm{QCD}}(r) = V_S(r) + \delta E_{\mathrm{US}}(r) + \dots , \tag{1}$$

$$\delta E_{\mathrm{US}}(r) = -i \frac{4\pi \alpha_s}{N_c} T_F \int_0^\infty dt\, e^{-i\Delta V(r) t} \langle \vec{r} \cdot \vec{E}^a(t) \varphi_{\mathrm{adj}}(t,0)^{ab} \vec{r} \cdot \vec{E}^b(0) \rangle , \tag{2}$$

where the dots denote higher order corrections in r; $\Delta V(r) = V_O(r) - V_S(r)$ denotes the difference between the octet and singlet potentials, which specifies the ultrasoft scale; $\vec{E}^a$ is the ultrasoft chromoelectric field. See Ref. [2] for details. In the following, we evaluate V_S and δE_{US} in perturbation theory especially using the large-β_0 approximation [11,12]. The corresponding diagrams are shown in Fig. 1.

Let us first sketch what we will do in the following. We introduce cutoff scales μ_1 and μ_2 to divide the energy region: $\Lambda_{\mathrm{QCD}} \ll \mu_2 \ll \Delta V \ll \mu_1 \ll r^{-1}$. We define V_S as a soft quantity by restricting the gluon momentum to be higher than μ_1. Similarly, we define δE_{US} as an ultrasoft quantity by requiring the relevant momentum p to be $\mu_2 < p < \mu_1$. Accordingly, we perform the multipole expansion as

$$V_{\mathrm{QCD}}(r) = V_S(r;\mu_1) + \delta E_{\mathrm{US}}(r;\mu_1,\mu_2) + \dots . \tag{3}$$

For the singlet potential $V_S(r;\mu_1)$, we follow the separation performed in Ref. [7][6]:

$$V_S(r;\mu_1) = V_S^{\mathrm{RF}}(r) + \mathcal{C}_2(\mu_1) r^2 + \mathcal{O}(r^3) , \tag{4}$$

where V_S^{RF} is a renormalon free part and has a Coulomb+linear form. The leading cutoff dependence in $V_S(r,\mu_1)$, $\mathcal{C}_2(\mu_1) r^2 \sim \mu_1^3 r^2$, is caused by the $u = 3/2$ renormalon. In this Letter, we show that $\delta E_{\mathrm{US}}(r;\mu_1,\mu_2)$ can be decomposed as

$$\delta E_{\mathrm{US}}(r;\mu_1,\mu_2) \sim \delta E_{\mathrm{US}}^{\mathrm{RF}}(r) - \mathcal{C}_2(\mu_1) r^2 + \mathcal{O}(\mu_2^4 r^3) , \tag{5}$$

where $\delta E_{\mathrm{US}}^{\mathrm{RF}}(r)$ is independent of μ_1 and μ_2 and is free from renormalons. As a result, in the multipole expansion (3), we have

$$V_{\mathrm{QCD}}(r) = V_S^{\mathrm{RF}}(r) + \delta E_{\mathrm{US}}^{\mathrm{RF}}(r) + \mathcal{O}(r^3) . \tag{6}$$

In this way, we can obtain a net contribution up to NLO, where each term does not have renormalon ambiguity.

[6] In Eq. (4), we omit the r-independent constant related to the $u = 1/2$ renormalon.