Physics Letters B 783 (2018) 253–262

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Conformal symmetry: Towards the link between the Fermi and the Planck scales

Mikhail Shaposhnikov [a], Andrey Shkerin [a,b,*]

[a] Institute of Physics, Laboratory for Particle Physics and Cosmology, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland
[b] Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary prospect 7a, 117312, Moscow, Russia

A R T I C L E I N F O

Article history:
Received 28 March 2018
Received in revised form 25 June 2018
Accepted 29 June 2018
Available online 3 July 2018
Editor: A. Ringwald

A B S T R A C T

If the mass of the Higgs boson is put to zero, the classical Lagrangian of the Standard Model (SM) becomes conformally invariant (CI). Taking into account quantum non-perturbative QCD effects violating CI leads to electroweak symmetry breaking with the scale $v \sim \Lambda_{QCD} \sim 100$ MeV which is three orders of magnitude less than it is observed experimentally. Depending on the mass of the top quark, the radiative corrections may lead to another minimum of the effective potential for the Higgs field with $v \gtrsim M_P$, where M_P is the Planck mass, at least 16 orders of magnitude more than it is observed. We explore yet another source of CI breaking associated with gravity. We suggest a non-perturbative mechanism that can reproduce the observed hierarchy between the Fermi and the Planck scales, by constructing an instanton configuration contributing to the vacuum expectation value of the Higgs field. The crucial role in this effect is played by the non-minimal coupling of the Higgs field to the Ricci scalar and by the approximate Weyl invariance of the theory for large values of the Higgs field.

1. Introduction and setup

The observed value of the Electroweak (EW) scale, represented by the Standard Model (SM) Higgs boson mass m_H, poses a challenge the physics beyond the SM must deal with. One aspect of this challenge comes from the fact that radiative corrections to m_H, engendered by the presence of degrees of freedom with some heavy mass scale (existing, for instance, in the standard GUTs – Grand Unified theories), shift it towards that scale. The apparent stability of the Higgs mass against such corrections requires either fine-tuning among the parameters of the theory or a special mechanism of their systematic suppression. The second aspect of the problem concerns with the smallness of the ratio of the EW scale to the GUT scale or to the Planck scale at which quantum gravity effects are expected to come into play. Combined together, the two issues are known as the hierarchy problem (for reviews see, e.g. [1,2]).

The first part of the hierarchy problem, related to the stability of the EW scale against perturbative quantum corrections, does not appear in theories containing light particles only,[1] for detailed arguments and previous references see [4–6]. Of course, even if one evades any heavy mass thresholds coming with the physics beyond the SM, the Planck scale enters unavoidably any theory comprising the SM and General Relativity (GR). As the Planck mass M_P does not represent a mass of any particle but rather serves as a parameter measuring the strength of gravitational interaction (for an overview see, e.g. a relevant chapter in [7]), the argument may remain in force in the presence of gravity as well [5,6,8].

It is tempting to use the conformal symmetry for solution of the second part of the hierarchy problem [9]. Indeed, since at the classical level the SM Lagrangian acquires a conformal invariance (CI) once m_H is put to zero, one can imagine to start from the conformally-invariant classical SM which has no EW symmetry breaking and generate the Higgs mass due to the CI violation.

One of the possible ways to generate the Higgs mass within the CI setting is associated with quantum conformal anomaly (see, e.g., [10]). Indeed, the UV regularisation of renormalizable field theories necessarily introduces a parameter with the dimension of mass, which violates CI at the quantum level and thus makes it to be

* Corresponding author.
E-mail addresses: Mikhail.Shaposhnikov@epfl.ch (M. Shaposhnikov), Andrey.Shkerin@epfl.ch (A. Shkerin).

[1] This requirement does not exclude Grand Unified Theories, which can be constructed without leptoquarks [3].

https://doi.org/10.1016/j.physletb.2018.06.068
0370-2693/

anomalous.[2] As a result, the effective potential for the Higgs field, accounting for higher-order radiative corrections, may develop a minimum displaced from the origin, potentially leading to the vacuum expectation value (vev) v of the Higgs field small compared with M_P or the GUT scale [13,14].

The Coleman–Weinberg (CW) scenario [13] in the SM can indeed be realised [15–17], but it leads to the Higgs and the top quark masses $m_H \simeq 7$ GeV and $m_t \lesssim 80$ GeV being far from those observed experimentally. If we take the physical values of dimensionless Higgs self-coupling λ and top quark Yukawa coupling y_t, the effective potential of the CI SM has a minimum around the QCD scale $\Lambda_{\text{QCD}} \sim 100$ MeV, associated with confinement and quark condensates [18], which is too far from the one realised in Nature. If the value of the top quark Yukawa coupling is smaller than some critical value, $y_t < y_{\text{crit}}$, this minimum is unique. For $y_t > y_{\text{crit}}$ yet another minimum is generated by the CW mechanism [19], with the very large vev $v \gtrsim M_P$, now many orders of magnitude larger that the EW scale. Due to experimental uncertainties it is not known yet whether y_t is larger or smaller than y_{crit} (for an overview see, e.g., [20]), but in any event the predictions of the CI SM are in sharp contrast with experiment.

In spite of this failure, the no-scale CI theories look very attractive and motivated many authors to search for different extensions of the SM, in which the mechanism may work and be phenomenologically acceptable. We mention just a few. The extended scalar sector was discussed on general grounds in [21], more recent works deal with the SM extended by right-handed neutrinos and a real scalar field [22], by an Abelian $B-L$ gauge field [23–25], or by non-abelian gauge groups [26,27].

All the considerations of the CI theories up to date were carried out without gravity. There is a clear rationale for this, based on (nearly scale-invariant) perturbation theory [28]: any *perturbative* corrections to the effective potential of the Higgs field coming from gravity are suppressed by the Planck mass [29], and in the absence of heavy particles they are numerically small. In the SM, the largest contribution is of the order $y_t^6 h^6/M_P^2$, which is negligible at the weak scale.

Clearly, the general difficulty of working with quantum gravity is that we are not aware of an explicit UV completed theory reducing to (CI) SM and GR at low energy scales. What one can do in this situation is to attribute to the unknown UV physics the properties which are found to be useful in resolving apparently low energy physic's questions. In particular, one can imagine that the Higgs mass arises *purely* due to some quantum gravity effects. Having accepted the CI setting, this could only imply the existence of a non-perturbative mechanism driving the Higgs field vev towards its observed value, some 17 orders of magnitude below M_P.

The aim of this paper is to argue that gravity can indeed generate in a non-perturbative way a new mass scale, many orders of magnitudes smaller than the Planck scale. Non-perturbative phenomena can manifest themselves in various ways. As one example, they can be associated with the strong coupling scale around which the content of the theory is reorganized, and, in particular, the physical degrees of freedom are rearranged.[3] Another possibility is the existence of euclidean classical configurations — the instantons — that contribute to correlation functions of the theory and may eventually result in drastic changes of the low energy observables.[4] In this paper our main concern will be with the second effect. In fact, it is not difficult to come to an idea that instantons or, more precisely, their large actions may somehow be involved in generating the hierarchy of scales. The smallness of the ratio v/M_P can be viewed as a result of an exponentially strong suppression of the Planck scale that generally appears in physical quantities as the only classical scale of the theory. Our intention is to use some simple yet explicit models comprising the Higgs and the gravitational fields as a playground in which the existence conditions for such instantons can be studied.

Of course, lacking a UV complete theory encompassing the SM and GR brings irremovable ambiguities in our analysis. Nevertheless, we argue that this ambiguity does not make quantitative investigation meaningless once it is clearly stated under what assumptions about the high energy behavior of the theory we work. We find this kind of approach quite appealing, as the line of reasoning can be reverted easily by claiming the observed value of v to be an argument in favour of those properties of the unknown theory, for which the working mechanism of generation of the Higgs mass is found.

Let us specify the framework in which we will work. Following the discussion above, we would like to exclude from consideration possible quantum (perturbative) corrections to the Higgs field vev, coming with the heavy mass thresholds associated with new physics. To this end, we require no degrees of freedom with the mass scales above the EW scale appear in the theory. That is, we demand the only classical dimensional parameter in the theory be the Planck mass. In this case, the Higgs-gravity sector of a theory under investigation, which we will be mainly interested in, is governed by the Horndeski construction or its extensions [32,33]. The vastness of possible models is further restricted by the requirement to reproduce the SM Higgs sector and GR at low energies and by the assumption that among higher-dimensional operators activating at high energies those are present that we find useful for the purpose of generating the hierarchy of scales.[5] It should be stressed that our goal here is to find an example of a model in which a mechanism of an exponential suppression of the Planck mass due to instantons exists. Hence, we do not have an intention to perform a survey of all possible models by studying effects from all possible higher-dimensional operators containing the Higgs and gravity fields. Nor do we intend to argue that a toy theory chosen to illustrate the mechanism can indeed be consistently embedded into the UV complete theory of gravity.

We will find that the crucial ingredient of the theory admitting the instantons with the desired properties is the non-minimal coupling of the Higgs field to the Ricci scalar. We will also find that the instantons generating the large hierarchy of scales favour the (approximate) Weyl invariance of the theory for large values of the scalar field.

The paper is organized as follows. In sec. 2 we introduce a simple model describing the dynamics of the gravitational and the classically massless scalar fields. We analyze euclidean classical configurations arising in this model, and discuss their possible influence on the vev of the scalar field. The results of the analysis motivate us to introduce certain modifications into the model. In sec. 3 we incorporate these modifications step by step. We find that the contribution of a certain classical configuration (the singular instanton) to the vev of the scalar field can actually make the latter non-zero and, at the same time, many orders of magnitude smaller than the Planck scale. In sec. 4 we apply our findings

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[2] If the requirement of renormalisability is removed, the theory can be made conformal at the quantum level [8,11,12]. We will consider such theories in a separate publication.

[3] For discussion of this possibility in context with the hierarchy problem see, e.g., [30].

[4] Perhaps, the most instructive example of this phenomenon is a discrete symmetry restoration in quantum mechanics of one dimension [31].

[5] The structure of theories at high energies can also be subject to constraints, e.g., by the requirement of asymptotic safety [34].