# Parton distribution amplitudes: Revealing correlations within the proton and Roper 

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## A R T I C L E IN F O

## Article history:

Received 23 February 2018
Received in revised form 12 June 2018
Accepted 28 June 2018
Available online 2 July 2018
Editor: J.-P. Blaizot

## Keywords:

Parton distribution amplitudes
Proton structure
Nonperturbative methods in quantum field theory
Dynamical chiral symmetry breaking
Light-front quantum field theory


#### Abstract

Constrained by solutions of the continuum three-valence-body bound-state equations, we use perturbation theory integral representations (PTIRs) to develop algebraic Ansätze for the Faddeev wave functions of the proton and its first radial excitation, delivering therewith a quantum field theory calculation of the pointwise behaviour of their leading-twist parton distribution amplitudes (PDAs). The proton's PDA is a broad, concave function, with its maximum shifted relative to the peak in QCD's conformal limit expression for this PDA. The size and direction of this shift signal the presence of both scalar and pseudovector diquark correlations in the nucleon, with the scalar generating around $60 \%$ of the proton's normalisation. The radial-excitation is constituted similarly, and the pointwise form of its PDA, which is negative on a material domain, is the result of marked interferences between the contributions from both types of diquark; particularly, the locus of zeros that highlights its character as a radial excitation. These features originate with the emergent phenomenon of dynamical chiral-symmetry breaking in the Standard Model.


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## 1. Introduction

Wave functions provide insights into composite systems, e.g. they express the presence and extent of correlations between constituents, and their signature in scattering processes; and thereby bridge experiment and theory. This is true within quantum chromodynamics (QCD), but there are difficulties. Everyday hadrons ( $p=$ proton, neutron, etc.) are constituted from up (u) and down (d) valence-quarks; but the Higgs boson generates current-masses for these fermions which are more than 100 -times smaller than the scale associated with the composite systems: $m_{u, d} \approx 2-4 \mathrm{MeV}$ cf. $m_{p} \approx 1 \mathrm{GeV}$. Plainly, the interaction energy greatly exceeds the rest masses of the anticipated constituents, making inapplicable the wave functions typical of Schrödinger quantum mechanics.

The difficulties appear chiefly because particle-number is not conserved by boosts; and severe challenges are faced when constituents are light, e.g. wave functions describing incoming and

[^0]outgoing scattering states then represent systems with different particle content, so a probability interpretation is lost. Such problems are circumvented by using a light-front formulation because eigenfunctions of the Hamiltonian are then independent of the system's four-momentum [1,2].

The light-front wave function of a hadron with momentum $P$ and spin $\lambda, \Phi(P, \lambda)$, is complicated. In terms of perturbation theory's partons, $\Phi(P, \lambda)$ has a countably-infinite Fock-space expansion. Were it necessary to use this complete object in analyses of even the simplest processes, then little connection between experiment and theory could be made. Fortunately, collinear factorisation in the treatment of hard exclusive processes entails that much can be gained merely by studying hadron leading-twist parton distribution amplitudes (PDAs) [3]. Such a PDA is obtained from the simplest term in the Fock-space expansion.

Regarding $S$-wave ground-state light-meson leading-twist PDAs, the last decade has seen real progress, not concerning their conformal limit [3]: $\varphi(x ; \zeta)=6 x(1-x), m_{\mathrm{p}} / \zeta \simeq 0$; but on $m_{p} / \zeta \simeq 1$, where they are now known to be broad, concave functions [4-12]. This resolves a longtime conflict, removing the possibility that such PDAs have a minimum at zero relative momentum [13].


Fig. 1. Poincaré covariant Faddeev equation. The shaded rectangle demarcates its kernel: single line, dressed-quark propagator; $\Gamma$, diquark correlation amplitude; and double line, diquark propagator. $\Psi$ is the Faddeev amplitude for a baryon of total momentum $P=\left(p_{1}+p_{2}\right)+p_{3}=p_{d}+p_{q}$. The wave function, $\chi$, is obtained by attaching the quark and diquark propagator legs to $\Psi$.

Concerning the proton's leading-twist PDA, however, the situation is as unsatisfactory today as it was previously for mesons. Estimates of low-order Mellin moments exist, obtained using sum rules [13,14] or lattice-QCD (lQCD) [15-17], but there are no quantum field theory computations of this PDA's pointwise behaviour; and nothing is known about the PDA of the proton's radial excitation.

## 2. Proton PDA: definition

In the isospin-symmetry limit, the proton possesses one independent leading-twist (twist-three) PDA [18], denoted $\varphi([x] ; \zeta)$ herein:

$$
\begin{align*}
& \langle 0| \varepsilon^{a b c} \tilde{u}_{+}^{a}\left(z_{1}\right) C^{\dagger} n u_{-}^{b}\left(z_{2}\right) \mathfrak{n} d_{+}^{c}\left(z_{3}\right)|P,+\rangle \\
& =: \frac{1}{2} i f_{p} n \cdot P \not n N_{+} \int_{0}^{1}[d x] \varphi([x] ; \zeta) \mathrm{e}^{-i n \cdot P \sum_{i} x_{i} z_{i}}, \tag{1}
\end{align*}
$$

where $n^{2}=0$; $(a, b, c)$ are colour indices; $q_{ \pm}=H_{ \pm} q:=(1 / 2)\left(\mathbf{I}_{\mathrm{D}} \pm\right.$ $\left.\gamma_{5}\right) q, \mathscr{n}=\gamma \cdot n ; \tilde{q}$ indicates matrix transpose; $C$ is the charge conjugation matrix, $N=N(P)$ is the proton's Euclidean Dirac spinor; $\int_{0}^{1}[d x] f([x])=\int_{0}^{1} d x_{1} d x_{2} d x_{3} \delta\left(1-\sum_{i} x_{i}\right) f([x])$; and $f_{p}$ measures the proton's "wave function at the origin".
$\varphi([x])$ can be computed once the proton's Poincaré-covariant wave function is in hand, viz.

$$
\begin{align*}
& \langle 0| \epsilon^{a b c} \tilde{u}^{a}\left(y_{1}\right) u^{b}\left(y_{2}\right) d^{c}\left(y_{3}\right)|P, \lambda\rangle= \\
& \quad \int \prod_{i=1}^{3}\left(\frac{\mathrm{~d}^{4} p_{i}}{(2 \pi)^{4}} e^{-i p_{i} y_{i}}\right) \delta\left(P-\sum_{i=1}^{3} p_{i}\right) \chi\left(p_{1}, p_{2}, p_{3}, P\right) . \tag{2}
\end{align*}
$$

Following thirty years of study [19-23], a clear picture has appeared. At an hadronic scale, the proton is a Borromean system, bound by two effects [24]: one originates in non-Abelian facets of QCD, expressed in the effective charge [25] and generating confined, nonpointlike but strongly-correlated colour-antitriplet diquarks in both the isoscalar-scalar and isotriplet-pseudovector channels; and that attraction is magnified by quark exchange associated with diquark breakup and reformation. The presence and character of the diquarks owe to the mechanism that dynamically breaks chiral symmetry in the Standard Model [24]. This understanding transforms the proton bound-state problem into that of solving the linear, homogeneous matrix equation in Fig. 1, which has been studied extensively, e.g. Refs. [24,26-31], so that the character of the solution is well known.

Recapitulating only essential features of the Faddeev equation solution herein, because extensive discussions are presented elsewhere, e.g. the appendices of Ref. [26], we recall that the proton Faddeev amplitude in Fig. 1 can be written:
$\Psi(P)=\psi_{1}+\psi_{2}+\psi_{3}$,
where the subscript identifies the bystander quark, i.e. the quark not participating in a diquark, $\psi_{3}$ gives $\psi_{1,2}$ by cyclic permutation of all quark labels, and
$\psi_{3}(\{p\},\{\alpha\},\{\sigma\})=\mathcal{N}_{3}^{0}+\mathcal{N}_{3}^{1}$,
$\mathcal{N}_{3}^{0}=\left[\Gamma^{0}(k ; K)\right]_{\sigma_{1} \sigma_{2}}^{\alpha_{1} \alpha_{2}} \Delta^{0}(K)[\mathcal{S}(\ell ; P) N(P)]_{\sigma_{3}}^{\alpha_{3}}$,
$\mathcal{N}_{3}^{1}=\left[\Gamma_{\mu}^{1 j}(k ; K)\right]_{\sigma_{1} \sigma_{2}}^{\alpha_{1} \alpha_{2}} \Delta_{\mu \nu}^{1}(K)\left[\mathcal{A}_{\nu}^{j}(\ell ; P) N(P)\right]_{\sigma_{3}}^{\alpha_{3}}$,
( $\{p\},\{\alpha\},\{\sigma\})$ are the momentum, isospin and spin labels of the dressed-quarks constituting the bound state; $P=p_{1}+p_{2}+p_{3}$ is the total momentum of the baryon; $k=p_{1}, K=p_{1}+p_{2}, \ell=-K+$ $(2 / 3) P$; and the $j$ sum runs over the $(1,1)=+1$ and $(1,0)=0$ isospin projections. The matrix-valued functions $\Gamma$ in Eqs. (4) are the diquark correlation amplitudes in Fig. $1 ; \Delta^{0}, \Delta_{\mu \nu}^{1}$ are the associated dressed-propagators; and $\mathcal{S}, \mathcal{A}_{\mu}^{j}$ are matrix-valued quarkdiquark amplitudes, describing the relative-momentum correlation between the diquark and bystander quark, viz. they are the objects returned by solving the Faddeev equation.

The proton's Faddeev wave function, $\chi$, is obtained from Eqs. (3), (4) by attaching the appropriate dressed-quark and -diquark propagators. All relevant quantities are known and we therefore proceed by using algebraic representations for every element, with each form and their relative strengths, when combined, based on the results of modern analyses [24,26-31]. The dressedquark and -diquark propagators are:

$$
\begin{align*}
& S(p)=(-i p+M) \sigma_{M}\left(p^{2}\right), \sigma_{M}(s)=1 /\left[s+M^{2}\right],  \tag{5a}\\
& \Delta^{0}(K)=\sigma_{M_{0}}\left(K^{2}\right), \Delta_{\mu \nu}^{1}(K)=T_{\mu \nu}(K) \sigma_{M_{1}}\left(K^{2}\right),  \tag{5b}\\
& \hat{\sigma}_{M}(s)=M^{2} \sigma_{M}(s) ; T_{\mu \nu}(K)=\left[\delta_{\mu \nu}-K_{\mu} K_{\nu} / K^{2}\right] ; \\
& n_{0} \Gamma^{0}(k ; K) C^{\dagger}=i \gamma_{5} \int_{-1}^{1} d z \rho(z) \hat{\sigma}_{\Lambda_{\Gamma}}\left(k_{+K}^{2}\right),  \tag{6a}\\
& n_{1} \Gamma_{\mu}^{1}(k ; K) C^{\dagger}=i\left(\gamma_{\mu}^{\mathrm{T}}+r_{1} f(k ; K)\left[k, \gamma_{\mu}^{\mathrm{T}}\right]\right) \\
& \quad \times \int_{-1}^{1} d z \rho(z) \hat{\sigma}_{\Lambda_{\Gamma}}\left(k_{+K}^{2}\right), \tag{6b}
\end{align*}
$$

where $\rho(z)=(3 / 4)\left(1-z^{2}\right), k_{+K}=k+(z-1) K / 2 ; \gamma_{\mu}^{\mathrm{T}}=T_{\mu \nu}(K) \gamma_{\nu}$, $f(k ; K)=k \cdot K /\left(k^{2} K^{2}(k-K)^{2}\right)^{1 / 2}$; and $r_{1}=1 / 4, n_{0,1}$ are fixed by requiring that the zeroth Mellin moment of the leading-twist PDA of each diquark correlation is $[n \cdot K / n \cdot P]$, i.e. correctly normalised. The final elements are:

$$
\begin{align*}
n \mathcal{S}(\ell ; P)= & i \int_{-1}^{1} d z \rho(z) \hat{\sigma}_{\Lambda_{p}^{0}}\left(w_{+P}\right),  \tag{7a}\\
n \mathcal{A}_{\nu}^{j}(\ell ; P)= & r_{\mathfrak{A}} \frac{1}{6} o^{j} \gamma_{5}\left[\gamma_{\nu}-i r_{P} P_{\nu}\right] \\
& \times \int_{-1}^{1} d z \rho(z) \hat{\sigma}_{\Lambda_{p}^{1}}\left(w_{+P}\right), \tag{7b}
\end{align*}
$$

where $w_{+P}=\left[-\ell_{+}+(2 / 3) P\right]^{2} ; o^{+}=\sqrt{ } 2, o^{0}=-1 ; r_{P}=13 / 87$; $r_{A}$ measures the relative $1^{+}: 0^{+}$diquark strengths in the Faddeev amplitude; and $n$ is that amplitude's canonical normalisation constant, whose value ensures the proton has unit charge [32].

We choose the parameters in Eqs. (6), (7) so as to emulate realistic Faddeev wave functions $[26,27,31,33]: M=2 / 5, M_{0}=2 / 3$,

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