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Constant temperature model for nuclear level density

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ABSTRACT

We study physics related to the nuclear level density calculated either in a realistic shell model or, equivalently, with the use of the statistical moments method. At excitation energy up to 12–15 MeV, the obtained level density grows exponentially being well described by the so-called constant temperature model. We discuss the physical meaning of the effective temperature parameter and its dependence on the interaction Hamiltonian including nucleon pairing and deformation effects. The possible interpretation relates the underlying physics with the gradual chaotization of typical wave functions rather than with the pairing phase transition.

1. Introduction

The importance of reliable knowledge of the nuclear level density is obvious for the understanding of nuclear reactions – in the laboratory, in technological applications, and in cosmos. There is a long history of various approaches to the problem of level density which cannot be fully reflected in this Introduction. We just briefly mention the Fermi-gas description [1–3], more advanced mean-field methods [4] accounting for the pairing correlations, and Monte-Carlo approaches [5]. Of course, modern shell-model (configuration interaction) theory in principle gives, for the accepted Hamiltonian, the exact result [6,7] limited in energy by the unavoidable truncation of the orbital space but this approach is always related to the diagonalization of prohibitively large matrices. The shell-model Monte-Carlo method [8] gives the results without diagonalization but it currently accounts only for the most regular parts of accepted interactions. It was earlier shown [9] that the interaction matrix elements corresponding to incoherent collision-like processes are equally contributing to the resulting level density increasing its width and providing its smooth energy dependence.

Modern versions of the shell model using the spectroscopically tested Hamiltonians allow us to predict reliably the level density up to excitation energy of about 12–15 MeV where the resonances in the continuum are still not too broad. This could be sufficient for many practical purposes. It turns out, see [9–11] and references therein, that frequently it is possible to avoid the full diagonalization using the so-called *moments method* [12] based on statistical properties of many-body wave functions. It was shown repeatedly that, in those cases which practically allow the complete diagonalization, results of the moments method are essentially identical at energies of interest to the results from the full shell-model solution. In such cases, the comparison with the experimental data becomes in fact a quality check for the underlying shell-model Hamiltonian.

In what follows we discuss some features of physics that determine the level density in a nucleus as an isolated quantum system of strongly interacting fermions. It turns out that in the majority of cases in the energy region of interest, that includes the beginning of the continuum where the energy levels become (still non-overlapping) resonances, the level density grows exponentially. A large systematics of experimental data for many nuclei and comparison with the Fermi-gas approaches can be found in Refs. [9,13,14]. The studies in the shell-model framework [9,11,15] seem to prefer the description usually associated with the so-called *constant temperature model* where the effective temperature T is introduced as an inverse coefficient in the exponent of the level density as a function of excitation energy. Varying the shell-model Hamiltonian we can determine the dependence of this parameter on various interaction parts. This puts limitations on the possible interpretation of the constant temperature model. In fact, the parameter T in the level density is rather an analog of the limiting Hagedorn temperature in particle physics [16–18]. The for-

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mally found *thermodynamic temperature* $T_{\rm t-d}$ starts at zero for the ground state and approaches the value of T at excitation energy E noticeably higher than T, usually around or above 10 MeV.

As in particle physics, this does not mean that the system cannot be heated further but it can be interpreted as a transition to the chaotic stage of more or less randomly interacting constituents. As known from many studies, the internal states of nuclei at such energy are close to random superpositions of many quasiparticle excitations so that the *local* structure of the spectra can be well juxtaposed to that of the Gaussian orthogonal ensemble [19,20]. At the same time, the obvious interpretation of this evolution as a phase transition from superfluid paired dynamics to the normal Fermi liquid is not sufficient. The specific examples below show that the behavior persists if the standard attractive pairing interaction (the source of nuclear superfluidity) is removed from the Hamiltonian or even substituted by repulsion. Supposedly we have to deal with a more general process of stochastization of dynamics as a typical feature of quantum many-body systems.

2. Constant temperature model

For all *sd*-nuclei, and all classes of states with different values of nuclear spin J, the level density was calculated and tabulated [11] using the shell model USDB Hamiltonian [21] and either the moments method or the full diagonalization; examples for heavier nuclei are given also in [9]. In the majority of cases, the resulting level density can be well described by the so-called *constant temperature formula*

$$\rho(E) = \rho_0 e^{E/T}, \tag{1}$$

where the prefactor is usually written in the form

$$\rho_0 = \frac{1}{T}\, e^{-E_0/T}. \tag{2}$$

Here we introduce two parameters, T and E_0, while E in eq. (1) is the actual nuclear excitation energy counted from the ground state. This parametrization was suggested long ago [22,17] and successfully used for the description of data [9,13–15,18]. In traditional Fermi-gas models, the level density typically grows as $\exp(\sqrt{2aE})$ with the constant a determined by the single-particle level density at the Fermi surface. Fig. 1 shows the evolution of the effective temperature T along the isotope chains of magnesium, aluminum, and silicon [11], while Fig. 2 illustrates the quality of description in the moments method and the USD B shell model Hamiltonian for ^{24}Mg when compared to the experimental level densities.

As discussed earlier [9,11], the parameter T of eq. (1) reaches its minimum at $N = Z$ or at neighboring odd nuclei. This quantity is kind of effective temperature kept constant within a broad interval of excitation energies E. This is the source of the name "constant temperature model", although the definition (1) just provides $1/T$ as the constant rate of increase of the level density as a function of excitation energy. As shown in [11], such a phenomenological expression is indeed working universally for almost all *sd*-nuclei described by the shell model. It provides the good description in the *pf*-region as well.

Here we have to stress that the effective temperature parameter T in eqs. (1) and (2) does not coincide with the temperature $T_{\rm t-d}$ found from thermodynamics for the system with the level density (1). Indeed, defining microcanonical thermodynamic entropy S through the cumulative level number

$$\mathcal{N}(E) = \int_0^E dE' \rho(E') = e^S, \tag{3}$$

we come to

$$S = \ln\left[\rho_0 T (e^{E/T} - 1)\right]. \tag{4}$$

As always for a system with a discrete energy spectrum, this expression violating the third law of thermodynamics acquires the meaning only at non-zero (practically quite small) excitation energy, when it makes sense to speak about the level density. Now we can introduce the thermodynamic temperature,

$$T_{\rm t-d} = \left(\frac{\partial S}{\partial E}\right)^{-1} = T\left(1 - e^{-E/T}\right), \tag{5}$$

which is always lower than our auxiliary temperature T but coincides with that at $E \gg T$. The thermodynamic temperature $T_{\rm t-d}$ starts from zero at very low excitation energy and then grows as a function of E to the maximum value of T, while the effective temperature T is constant in the broad interval of excitation energies, usually including the continuum threshold. The thermodynamic heat capacity $\partial E/\partial T_{\rm t-d} = \exp(E/T)$ increases from $E = 0$ exponentially (in usual Fermi-gas models it grows linearly).

As known from discussions of the Hagedorn temperature extracted from the exponentially growing density of resonances, eq. (5) does not mean the existence of the absolute hottest temperature. The system just becomes a chaotic gas of randomly interacting constituents (quarks or strings in quantum field theory and quasiparticles in the nuclear case). At higher excitation energy the exponential level density law (1) does not work anymore; the partition function defined in a standard way,

$$\mathrm{Tr}(e^{-H/T_{\rm t-d}}) = \int dE\, \rho(E) e^{-E/T_{\rm t-d}}, \tag{6}$$

would diverge. It was shown long ago, see for example [19], that the full shell-model level density in a finite fermionic space is given by a particle–hole symmetric bell-shape curve that is essentially Gaussian close to the centroid; the going down part beyond the energy centroid formally corresponds to negative temperature (inversion of occupancies in the finite Hilbert space). Above some energy, realistically much lower than the Gaussian centroid, the validity of the description (1) expires, even if the states outside of the originally truncated orbital space still do not enter the game. From Fig. 3, one can see the excitation energy limits (~ 15 MeV) for the constant temperature model applied to ^{24}Mg.

Assuming for the global shell-model level density the standard Gaussian shape,

$$\rho_g(E) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(E-E_c)^2/(2\sigma^2)}, \tag{7}$$

we should be able to match continuously this function with the constant temperature model valid at low excitation energy. The global description of the Gaussian (7) introduces [19] an average temperature

$$T_g(E) = \frac{\sigma^2}{E_c - E}. \tag{8}$$

This shows the infinite temperature at the centroid with the jump to negative temperature after the middle which physically displays the particle–hole symmetry in a finite orbital space.

As shown long ago [19,23], the formally defined global temperature T_g with its Gaussian energy dependence agrees with the temperature parameter fit by the fermionic occupation numbers of *individual* states found in the exact shell-model solution (see also the discussion in [24] combining atomic and nuclear examples). This means that, starting with some excitation energy, the