Physics Letters B ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Anomalous transport model with axial magnetic fields

Karl Landsteiner [a], Yan Liu [b]

[a] Instituto de Física Teórica UAM/CSIC, C/ Nicolás Cabrera 13-15, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
[b] Department of Space Science, and International Research Institute of Multidisciplinary Science, Beihang University, Beijing 100191, China

ARTICLE INFO

Article history:
Received 12 November 2017
Received in revised form 16 March 2018
Accepted 30 April 2018
Available online xxxx
Editor: N. Lambert

ABSTRACT

The transport properties of massless fermions in 3 + 1 spacetime dimension have been in the focus of recent theoretical and experimental research. New transport properties appear as consequences of chiral anomalies. The most prominent is the generation of a current in a magnetic field, the so-called chiral magnetic effect leading to an enhancement of the electric conductivity (negative magnetoresistivity). We study the analogous effect for axial magnetic fields that couple with opposite signs to fermions of different chirality. We emphasize local charge conservation and study the induced magneto-conductivities proportional to an electric field and a gradient in temperature. We find that the magnetoconductivity is enhanced whereas the magneto-thermoelectric conductivity is diminished. As a side result we interpret an anomalous contribution to the entropy current as a generalized thermal Hall effect.

1. Introduction and motivation

Chiral anomalies [1,2] and the specific transport phenomena induced by them such as the chiral magnetic and the chiral vortical effects have been extensively discussed in the recent years (see [3,4] for reviews).

In a theory of massless Dirac fermions the vector current $J^\mu = \bar\Psi\gamma^\mu\Psi$ and axial current $J_5^\mu = \bar\Psi\gamma_5\gamma^\mu\Psi$ can be defined. In such a theory the chiral magnetic effect (CME) describes the generation of an electric current in a magnetic field in the presence of an axial chemical potential

$$\vec J = \frac{\mu_5}{2\pi^2}\vec B\,, \tag{1}$$

where μ_5 is the axial chemical potential conjugate to the axial charge operator $Q_5 = \int d^3x\bar\Psi\gamma_5\gamma^0\Psi$.

This formula has to be interpreted with care. At first sight it predicts the generation of a current in equilibrium. It has been pointed out however that such an equilibrium current is forbidden by the so-called Bloch theorem. In relation to the CME this theorem has first been invoked in a condensed matter context in [5]. A recent discussion of the Bloch theorem has been given in [6]. The theorem can be formulated as

$$\int d^3x\,\vec J(x) = 0\,, \tag{2}$$

in thermal equilibrium. Seemingly this is violated by eq. (1) for a homogeneous magnetic field. The important point emphasized in [6] is that the Bloch theorem is valid only for exactly conserved currents. This allows to resolve the tension between eq. (1) and the Bloch theorem. More precisely eq. (1) holds only for the so-called covariant version of the current. This covariant current is not a truly conserved current but rather has the anomaly

$$\partial_\mu J^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F^5_{\rho\lambda}\,, \tag{3}$$

where one also introduces a axial field A^5_μ as source for insertions of the axial current J^μ_5. Similarly the covariant version of the axial anomaly is

$$\partial_\mu J^\mu_5 = \frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\lambda}\left(F_{\mu\nu}F_{\rho\lambda} + F^5_{\mu\nu}F^5_{\rho\lambda}\right)\,. \tag{4}$$

In quantum field theory the currents are composite operators and have to be regularized. This regularization introduces certain ambiguities that have to be fixed by demanding certain classical properties of the currents to hold on the quantum level. One way to fix these ambiguities is to define J^μ and J^μ_5 to be invariant objects under both vector- and axial-type gauge transformations [7]. The disadvantage of this definition is that it does not result in a conserved vector like current but rather leads to the anomaly eq. (3).

E-mail addresses: karl.landsteiner@csic.es (K. Landsteiner), yanliu@buaa.edu.cn (Y. Liu).

https://doi.org/10.1016/j.physletb.2018.04.068

On the other hand one can insist on the vector like current to be exactly conserved $\partial_\mu \mathcal{J}^\mu = 0$. The relation between the two definitions of currents is

$$\mathcal{J}^\mu = J^\mu - \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_\nu F_{\rho\lambda}\,. \tag{5}$$

Due the axial anomaly the axial vector J_5^μ is never conserved and therefore its source A^5_μ can not be interpreted as a true gauge field. Therefore the Chern–Simons current in (5) is a physical current in a completely analogous way as the Chern–Simons current appearing in the quantum Hall effect. This resolves the tension between the chiral magnetic effect and the Bloch theorem in the following manner. Thermal equilibrium is defined by the grand canonical ensemble with density matrix $\exp(-(H-\mu_5 Q_5)/T)$. This is equivalent to considering the theory in the background of a temporal component of the axial field $A_0^5 = \mu_5$. Now the chiral magnetic effect in the exactly conserved current $\mathcal{J}^\mu$ takes the form [8]

$$\vec{\mathcal{J}} = \frac{\mu_5}{2\pi^2}\vec{B} - \frac{A_0^5}{2\pi^2}\vec{B}\,, \tag{6}$$

where the second term stems from the Chern–Simons current in eq. (5). Since in strict equilibrium $A_0^5 = \mu_5$ this shows that the chiral magnetic effect for the conserved current (5) vanishes as demanded by the Bloch theorem. The importance of defining the coserved current has also been discussed in chiral kinetic theory in [9].

On the other hand the closely related chiral separation effect

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B}\,, \tag{7}$$

does not suffer any such correction. Since the axial current is always affected by an anomaly there is no contradiction to the Bloch theorem as pointed out in [6].

There is however a third related effect if one allows for axial magnetic fields, $\vec{B}_5 = \vec{\nabla}\times\vec{A}_5$. This is a magnetic field that couples with opposite signs to fermions of different chirality. The axial magnetic effect takes the form

$$\vec{J} = \vec{\mathcal{J}} = \frac{\mu}{2\pi^2}\vec{B}_5\,. \tag{8}$$

Formally it describes the generation of a vector-like current in the background of an axial magnetic field at finite (vector-like) chemical potential. Note that the formula holds for both the covariant and the conserved form of the currents. Therefore this formula seems to be in much greater tension with the Bloch theorem than the chiral magnetic effect. One might dismiss this tension on the grounds that so far at a fundamental level no axial fields seem to exist in nature. However, it has been argued that such fields can appear in the effective description of the electronics of advanced materials, the so-called Weyl semimetals [10–13]. A low energy field theoretical description of the electronics of these materials given by the Dirac equation

$$\gamma^\mu(iD_\mu + b_\mu\gamma_5)\Psi = 0\,. \tag{9}$$

Here D_μ is the usual covariant derivative and the parameter b^μ enters just like the field A^5_μ coupling to the axial current. It has been argued that straining such materials can lead to spatial variation of the parameter b^μ and in consequence to the appearance of effective axial magnetic fields in eq. (9). The reason why there is no contradiction to the Bloch theorem in this case is as follows. The parameter b^μ exists only within the material and necessarily vanishes outside. If for definiteness we assume the axial magnetic field to be directed along the z direction and we compute the total axial flux at through a surface Ω at some fixed $z = z_0$

$$\Phi_5 = \int_\Omega dxdy\, B^5_z(x,y,z_0) = \int_{\partial\Omega} d\vec{S}\cdot\vec{b} = 0\,, \tag{10}$$

since one can always take the boundary of the surface to lie entirely outside the material where $\vec{b} = 0$. Therefore the axial analogue of the chiral magnetic effect (8) can not induce a net current and this resolves the tension with the Bloch theorem since no net current can be generated [14,15].

We will take these considerations as motivation to study electro- and thermo-magnetotransport in the background of axial magnetic fields under the assumption that the Bloch theorem is implemented by a vanishing net axial magnetic flux (10). This implies that the net equilibrium electric current vanishes but as we will see upon applying an electric field (or equivalently a gradient in chemical potential) and a temperature gradient leads to anomaly induced net contributions to the currents.

2. Anomalous transport

We study a simple of model of anomalous transport with coupled energy and charge transport. This means that in contrast to a full hydrodynamic model we assume that no significant collective flow parametrized by a flow velocity develops.[1] Not only is this a simpler model allowing to study the effects of anomalies on transport it might also be more directly relevant to systems where elastic scattering on impurities impedes the build up of collective flow.

We develop now a formal transport model based on the anomalous continuity equations

$$\dot{\epsilon} + \vec{\nabla}\cdot\vec{J}_\epsilon = \vec{E}\cdot\vec{J}\,, \tag{11}$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{J} = c\vec{E}\cdot\vec{B}\,, \tag{12}$$

where ϵ is the energy density and $\vec{J}_\epsilon$ is the energy current. Charge conservation is affected by an anomaly with anomaly coefficient c. The right hand side of equation (11) quantifies the energy injected into the system by an electric field (Joule heating) whereas (12) describes the (covariant) anomaly. So far this is not specific to axial magnetic fields but rather relies only on the presence of an anomaly in the current $J^\mu = (\rho, \vec{J})$.

To discuss transport we write down constitutive relations for $\vec{J}_\epsilon$, $\vec{J}$ and take as thermodynamic forces the gradients in the thermodynamic potentials and external electric and magnetic fields,

$$\begin{pmatrix} \vec{J}_\epsilon \\ \vec{J} \end{pmatrix} = L\cdot\begin{pmatrix} \vec{\nabla}\left(\frac{1}{T}\right) \\ \frac{\vec{E}}{T} - \vec{\nabla}\left(\frac{\mu}{T}\right)\end{pmatrix} + \begin{pmatrix}\hat{\sigma}_B \\ \sigma_B\end{pmatrix}\vec{B}\,. \tag{13}$$

The matrix L encodes response due to gradients in chemical potential and temperature. $\{\hat{\sigma}_B, \sigma_B\}$ describe response due to the magnetic field. In principle we could also allow an independent response due to the electric field. In our ansatz we have thus anticipated that positivity of entropy production is not compatible with such additional terms in the constitutive relations.

[1] This does not mean that the velocity or the variation of the velocity is zero, just that it cannot be determined by the conserved equations. Our transport model can not be obtained from hydrodynamics by setting the flow velocities to zero. Hydrodynamic flow (i.e. non vanishing velocity) appears already at zeroth order in derivatives and this imposes constraints on the first order transport coefficients that can appear in the constitutive relations [16]. Since for strong momentum relaxation flow is absent such relations are not present. This model has similarity to the treatment in the theory for incoherent metal in 2+1D [17].