Physics Letters B 780 (2018) 29–33

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Hydrogen-like spectrum of spontaneously created brane universes with de-Sitter ground state

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ARTICLE INFO

Article history:
Received 9 December 2017
Accepted 16 January 2018
Available online 31 January 2018
Editor: M. Cvetič

ABSTRACT

Unification of Randall–Sundrum and Regge–Teitelboim brane cosmologies gives birth to a serendipitous Higgs–deSitter interplay. A localized Dvali–Gabadadze–Porrati scalar field, governed by a particular (analytically derived) double-well quartic potential, becomes a mandatory ingredient for supporting a deSitter brane universe. When upgraded to a general Higgs potential, the brane surface tension gets quantized, resembling a Hydrogen atom spectrum, with deSitter universe serving as the ground state. This reflects the local/global structure of the Euclidean manifold: From finite energy density no-boundary initial conditions, via a novel acceleration divide filter, to exact matching conditions at the exclusive nucleation point. Imaginary time periodicity comes as a bonus, with the associated Hawking temperature vanishing at the continuum limit. Upon spontaneous creation, while a finite number of levels describe universes dominated by a residual dark energy combined with damped matter oscillations, an infinite tower of excited levels undergo a Big Crunch.

1. Introduction

The no-boundary proposal [1] invokes basic quantum mechanics to avoid the classically unavoidable Big Bang singularity. Creation in this language is a smooth Euclidean to Lorentzian transition, with the emerging (finite scale factor) universe resembling alpha decay. The simplest model of this kind is constructed at the level of the mini superspace, requires a positive cosmological constant $\Lambda > 0$, and can only be implemented for a closed $k > 0$ space. A variant which introduces a supplementary embryonic era can be realized, ad-hoc [2] by including a radiation energy density term, field theoretically by invoking the embedding approach [3], or via the landscape of string theory [4]. Brane extensions have also been discussed [5]. The theoretical highlight of the no-boundary proposal is the wave function of the universe, the solution of the Schrodinger Wheeler–deWitt (WdW) equation [6].

The two Randall–Sundrum (RS) models [7], followed by their Dvali–Gabadadze–Porrati (DGP) and Collins–Holdom (CH) extensions [8] which supplement a 4-dim Einstein–Hilbert part to the underlying 5-dim action, are presumably the prototype brane models. The first rights are reserved, however, to the Regge–Teitelboim (RT) model [9] where the universe is treated as a 4-dim extended test object floating geodesically [10] in a 5-dim non-dynamical background. Moreover, the first field theoretically consistent brane variation, albeit in a flat spacetime, was formulated by Dirac [11]. Exporting the Dirac prescription to the gravitational regime [12] allows us to treat the variety of models as special limits of a single unified brane cosmology. This Letter attempts to take the no-boundary proposal one step further to expose the Hydrogen-like spectrum (with deSitter as the ground state) of spontaneously created unified brane universes.

2. Unified brane cosmology in a nutshell

Let the 4-dim FLRW cosmological line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \quad (1)$$

be isometrically embedded within a Z_2-symmetric (L and R branches, respectively) 5-dim AdS background characterized by a negative cosmological constant $\Lambda_5 < 0$. This can be done for any scale factor $a(t)$ and without imposing any geometrical constraints. The associated extrinsic curvatures are given explicitly by

$$\mathcal{K}^{L,R}_{\mu\nu} = \begin{bmatrix} \frac{1}{\xi}\left(\frac{\ddot{a}}{a} - \frac{1}{6}\Lambda_5\right) & 0 \\ 0 & -\xi a^2 \gamma_{ij}(r,\theta) \end{bmatrix}. \quad (2)$$

It is $\xi(a)$ which governs the cosmic evolution equation, with the latter cast into the familiar FLRW format

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https://doi.org/10.1016/j.physletb.2018.01.044

$$\frac{\dot{a}^2+k}{a^2}=\frac{\Lambda_5}{6}+\xi^2(a)\,. \tag{3}$$

Within the framework of unified brane cosmology [12], in a nutshell, $\xi(a)$ is the root of the cubic equation

$$\rho=\frac{3\xi^2}{8\pi G_4}+\frac{3\xi}{4\pi G_5}+\frac{\Lambda_5}{16\pi G_4}+\frac{\omega}{\sqrt{3}\xi a^4}\,. \tag{4}$$

$G_{4,5}$ denote the 4,5-dim gravitational constants, respectively, and $\rho(a)$ stands for the localized DGP energy density on the brane. No specific equation of state $P=P(\rho)$ has been assumed. The ω-term (ω is a conserved charge), resembles (but not to be confused with) the dark radiation term which is known to accompany RS cosmology, is the fingerprint of the underlying RT model. It owes its existence to the built-in integrability of the brane's geodesic equations of motion. The special limits include:

- DGP limit ($\omega=0$): The now quadratic eq. (4) admits [13] two branches $\xi_\pm(a)$.
- RS limit ($\omega=0, G_4\to\infty$): $\xi_+(a)$ becomes proportional to $\rho(a)$, so that the FLRW equation is unconventionally sourced [14] by $\rho_{total}=\frac{\Lambda_5}{2}+\frac{1}{3}(4\pi G_5\rho)^2$.
- GR limit ($\omega=0, G_5\to\infty$): Λ_5 simply decouples.
- RT limit ($\omega\neq 0, G_5\to\infty$): The bulk is kept non-dynamical, $\Lambda_5\neq 0$ is optional. Sticking to the original FLRW format, one formally replaces ρ by $\rho_{total}=\rho+\rho_d$, compactly squeezing the entire deviation from GR into an effective 'dark' component $\rho_d(\rho)$. The latter must of course vanish for $\omega=0$, obeying

$$\rho_d{}^2\left(8\pi G_4(\rho+\rho_d)-\frac{\Lambda_5}{2}\right)=\frac{\omega^2}{a^8}\,. \tag{5}$$

- In the general case [12], one may follow the formalism specified by eq. (5), only with modified $\{\rho^\star,\rho_d^\star\}$ replacing $\{\rho,\rho_d\}$, where $\rho^\star=\rho-3\xi/4\pi G_5$.

3. Higgs ↔ deSitter interplay

We start with a deceptively naive question: *What are the field theoretical ingredients necessary for supporting a deSitter brane?* It is well known that, within the framework of GR, introducing a positive cosmological constant $\Lambda_4>0$ will do. However, once a non-trivial $\rho_d(\rho)$ enters the game, the answer is not straight forward any more. Our goal is to end up with a constant $\xi(a)$. Hence, the way to cancel out the ω-term in eq. (4) is to arrange for a suitable energy density

$$\rho(a)=\sigma+\frac{\omega}{a^4\sqrt{\Lambda_4-\frac{1}{2}\Lambda_5}}\,. \tag{6}$$

We are after a tenable field theoretical action capable of (i) Sourcing the above radiation term, (ii) Fixing the otherwise arbitrary ω-charge, and (iii) Bypassing fine tuning. This can be achieved by introducing a DGP brane localized real scalar field $\phi(x)$, subject to a particular uniquely prescribed scalar potential $V(\phi)$.

The idea is to parametrically express the scalar potential $V(\phi)=\rho-\frac{1}{2}\dot{\phi}^2$ and its gradient $V'(\phi)=-\ddot{\phi}-3\frac{\dot{a}}{a}\dot{\phi}$ as explicit functions of a, and then convert these two expressions into a single a-independent differential equation. We take advantage of the constant value of $\xi(a)^2=\frac{1}{3}(\Lambda_4-\frac{1}{2}\Lambda_5)$ to first prepare $\rho(a)$ and $\rho'(a)$ (using Eq. (4)), and $\dot{a}/a$ (using Eq. (3)) as functions of a. With this in hand, one can further express $\dot{\phi}^2=\rho(a)+\frac{1}{6}a\rho'(a)$ and $\ddot{\phi}=-\frac{\dot{a}}{6\dot{\phi}}(a\rho'(a))'$, and eventually target $V(\phi)$ and $V'(\phi)$, as functions of the a. Crucial for our discussion is the non-linear differential equation that emerges upon the elimination of the a-parameter. It reads

$$\frac{1}{4}W'(\phi)^2=-k\sqrt{\frac{3}{\omega}\sqrt{\Lambda_4-\frac{1}{2}\Lambda_5}W(\phi)^{\frac{3}{2}}+\frac{\Lambda_4}{3}W(\phi)}\,, \tag{7}$$

where $W(\phi)=V(\phi)-\sigma$. Counter intuitively, the exact analytic solution is surprisingly familiar

$$\boxed{V(\phi)=\sigma+\lambda^2\left(\phi^2-v^2\right)^2} \tag{8}$$

A restricted Higgs potential has made its appearance

$$\lambda^2=\frac{3k^2}{16\omega}\sqrt{\Lambda_4-\frac{1}{2}\Lambda_5}\,,\ \lambda^2v^2=\frac{\Lambda_4}{12}\,, \tag{9}$$

$$\sigma(\lambda,v)=\frac{\Lambda_4}{8\pi G_4}+\frac{\sqrt{3}}{4\pi G_5}\sqrt{\Lambda_4-\frac{1}{2}\Lambda_5}\equiv\sigma_0\,. \tag{10}$$

It consistently generalizes the special RT special case [15]. Note that the Higgs potential is a necessary but not a sufficient ingredient for supporting a de-Sitter brane. Since $3a^4\sqrt{\Lambda_4-\frac{1}{2}\Lambda_5}\dot{\phi}^2=4\omega$, the initial value $\dot{\phi}_c$, required by the 2nd order differential KG equation, gets fixed by the initial scale factor value a_c.

The classical solution of the field equations is given by

$$a(t)=\sqrt{\frac{3k}{\Lambda_4}}\cosh\sqrt{\frac{\Lambda_4}{3}}t\,,\phi(t)=v\tanh\sqrt{\frac{\Lambda_4}{3}}t\,. \tag{11}$$

The symmetry role played by the so-called proper scalar field $b(t)=a(t)\phi(t)/v$ is manifest via the equilateral hyperbola $a(t)^2-b(t)^2=3k/\Lambda_4$. Note that, in the spherically symmetric representation, the deSitter spacetime is counter intuitively accompanied by a *non-singular* time dependent 'slinky' ($\phi\equiv 0$ on the horizon) scalar hair [15], thereby avoiding the no-hair theorems of GR.

4. Variant Euclidization

Performing a Wick rotation $t\to i(\tau-\tau_c)$ implies

$$a(t)\to\alpha(\tau)=\sqrt{\frac{3k}{\Lambda_4}}\cos\sqrt{\frac{\Lambda_4}{3}}(\tau-\tau_c)\,, \tag{12}$$

$$\phi(t)\to i\chi(\tau)=iv\tan\sqrt{\frac{\Lambda_4}{3}}(\tau-\tau_c)\,, \tag{13}$$

normalized such that $\alpha(0)=0$, constituting a circle in the Euclidean phase plane $\alpha(\tau)^2+\beta(\tau)^2=3k/\Lambda_4$ (see Fig. 3), where $b(t)\to i\beta(\tau)$. Globally, the de-Sitter imaginary time periodicity $\Delta\tau=2\pi\sqrt{3/\Lambda_4}$ is now clearly manifest.

Had $a(t)\to\alpha(\tau)$ been conventionally accompanied by $\phi(t)\to\chi(\tau)$, the KG τ-evolution in the Euclidean regime would have been governed [16] by $V_E(\chi)=-V(\chi)$. This in turn would give rise to the well known upside-down potential $V_E(\chi)=-\sigma-\lambda^2(\chi^2-v^2)^2$. However, in the present case $a(t)\to\alpha(\tau)$ is unconventionally accompanied by $\phi(t)\to i\chi(\tau)$ and hence $b(t)\to i\beta(\tau)$, so the rules of the game are changed dramatically. A closer inspection reveals that the KG equation, while keeping its generic form, is actually being governed by $V_E(\chi)=+V(i\chi)$, translated in our case into

$$\boxed{V_E(\chi)=\sigma+\lambda^2(\chi^2+v^2)^2} \tag{14}$$

notably abandoning the upside down double well shape.

The scalar potential in the Lorentzian regime and its companion in the Euclidean regime are given by eqs. (8), (14), respectively.