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Redshift and lateshift from homogeneous and isotropic modified dispersion relations

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ABSTRACT

Observables which would indicate a modified vacuum dispersion relations, possibly caused by quantum gravity effects, are a four momentum dependence of the cosmological redshift and the existence of a so called lateshift effect for massless or very light particles. Existence or non-existence of the latter is currently analyzed on the basis of the available observational data from gamma-ray bursts and compared to predictions of specific modified dispersion relation models. We consider the most general perturbation of the general relativistic dispersion relation of freely falling particles on homogeneous and isotropic spacetimes and derive the red- and lateshift to first order in the perturbation. Our result generalizes the existing formulae in the literature and we find that there exist modified dispersion relations causing both, one or none of the two effects to first order.

1. Introduction

Most information about the properties of gravity are obtained by probing the geometry of spacetime through the observation of freely falling particles. In order to observe traces of the expected quantum nature of the gravitational interaction, one option is to look for their manifestation in the propagation of particles through spacetime, which we observe with telescopes. The theoretical prediction of such effects is one branch of quantum gravity phenomenology [1]. The pictorial idea why quantum gravity effects may become visible in this way is the following. Test particles probe spacetime on length scales which are inverse proportional to their energy. Thus the higher the energy of the particles, the smaller the length scale probed. Quantum gravity effects are expected to become relevant at the Planck scale and hence particles with energies closer to the Planck energy E_{pl} should interact stronger with the quantum nature of gravity than lower energetic ones. Therefore, the propagation of high energetic particles through spacetime may deviate from their predicted behavior by classical general relativity. Since the energy of a particle is observer dependent this pictorial idea needs to be formulated more precisely in terms of the particle's four momentum, instead of its energy, what we will do during the derivations of this letter.

As long as a fundamental theory of quantum gravity is not available to predict this effect from the scattering between gravitons and the probe particles such quantum gravity effects can be modeled phenomenologically by a modification of the relativistic dispersion relation of freely falling point particles, see [2–12] and references therein.

Even though the particles we observer have energies below the Planck energy, the small effect may accumulate over a long particle travel time and become detectable. In particular observations from high redshift gamma-ray bursts (GRBs) are candidates to find traces of Planck scale induced modified dispersion relations (MDR) [13–16]. One most prominent signature would be a so called lateshift observation [17], i.e. an advance or a delay in the expected time of arrival of high energetic photons and neutrinos from the same source compared to low energetic ones emitted at the same time. Recently a preliminary analysis of the ICECUBE data for such a lateshift has been performed in [18] as well as an analysis of GRBs detected with the Fermi Gamma-Ray Space Telescope [19–21].

To deduce a MDR from the measured time of arrival data of neutrinos and photons from GRBs a derivation of the lateshift effect from a most general modification of the general relativistic dispersion relation is required. Usually specific models are assumed and the lateshift is derived for these classes of MDRs [1, 14,15,22–24].

In this letter we derive the redshift and lateshift from an arbitrary perturbation of the general relativistic dispersion relation to first order in the perturbation. Observation or not-observation of a modified redshift or a lateshift effect then directly leads to condi-

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tions the perturbation of the dispersion relation must satisfy to be viable. As an interesting insight from the general red- and lateshift formula we findMDRs which predict both aforementioned effects, only one of them or even none to first order.

2. Dispersion relations as Hamilton functions on spacetime

To derive the lateshift from the dispersion relation of point particles on spacetime we interpret a dispersion relation as level sets of a Hamilton function on the spacetime's cotangent bundle, as it turned out to be a very useful framework to treat MDRs on curved spacetimes covariantly [25–27].

The four momentum of a particle is a 1-form P on spacetime which can be expanded in local coordinates around a point x as $P = p_a dx^a$. The tuple (x, p) denotes the particle's momentum p at the spacetime position x. A dispersion relation is a level set of a Hamilton function $H(x, p)$ which determines the particle's motion. This covariant formulation of dispersion relations on curved spacetimes has the advantage that it allows to study dispersion relations on the basis of the particle's four momentum without referring to the observer dependent notion of a particle's energy or spatial momentum.

Homogeneous and isotropic dispersion relations are characterized by Hamilton functions with a specific dependence on the particle's positions and momenta. As shown in [26] the most general homogeneous and isotropic dispersion relation is given by the level sets of the Hamiltonian

$$H(x,p) = H(t, p_t, w), \quad w^2 = p_r^2\chi^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\sin^2\theta}\,, \tag{1}$$

where $\chi = \sqrt{1-kr^2}$. Due to the high symmetry the Hamilton equations of motion, which determine the propagation of the particle through spacetime, can partly be solved and reduce to

$$\begin{aligned} &\dot p_t = -\partial_t H, \quad p_r = \frac{K_1}{\chi}, \quad p_\theta = 0, \quad p_\phi = 0,\\ &\dot t = \partial_{p_t} H, \quad \dot r = \partial_w H \frac{1}{w}\chi K_1, \quad \theta = \frac{\pi}{2}, \quad \phi = 0\,, \end{aligned} \tag{2}$$

where $K_1^2 = w^2$ is a constant of motion.

3. The perturbed dispersion relation

The most general perturbation of the homogeneous and isotropic general relativistic dispersion relation is given by the level sets of

$$H(t, p_t, w) = -{p_t}^2 + a(t)^{-2} w^2 + \epsilon h(t, p_t, w)\,. \tag{3}$$

The perturbation $h(t, p_t, w)$ can be an arbitrary function of t, p_t and w, and ϵ is an arbitrary perturbation parameter. In the context of quantum gravity or Planck scale induced perturbations it may be identified with the Planck scale, while other sources of a modification of the dispersion relation may require a different perturbation parameter. For the calculations below we do not fix the origin of the perturbation.

To derive the redshift and lateshift from (3) we use the Hamilton equations of motion

$$\dot t = -2p_t + \epsilon\partial_{p_t} h\,, \quad \dot r = \chi\left(\frac{2w}{a^2} + \epsilon\partial_w h\right), \tag{4}$$

and the dispersion relation

$$-{p_t}^2 + a^{-2}w^2 + \epsilon h(t, p_t, w) = -m^2\,. \tag{5}$$

The time dependence of the scale factor a will from now on only be displayed when necessary.

3.1. Redshift

The dispersion relation (5) determines p_t as function of t, r and w without solving any equation of motion. From the ansatz $p_t = p_t^0 + \epsilon p_t^1$ one easily finds

$$p_t(t, w, m) = -\sqrt{m^2 + \frac{w^2}{a^2}} + \epsilon\frac{h(t, p_t^0(t, w, m), w)}{2p_t^0(t, w, m)}\,, \tag{6}$$

and thus for massless particles

$$p_t(t, w, 0) = -\frac{w}{a} - \epsilon\frac{a}{2w} h(t, p_t^0(t, w, 0), w)\,. \tag{7}$$

The redshift of a photon which is emitted at time t_i with a coordinate time-momentum $p_t(t_i, w) = p_t(t_i, w, 0)$ and observed at time t_f with coordinate momentum $p_t(t_f, w) = p_t(t_f, w, 0)$, subject to the dispersion relation in consideration then is

$$\begin{aligned} z(t_i, t_f) &= \frac{p_t(t_i, w)}{p_t(t_f, w)} - 1\\ &= \left(\frac{a(t_f)}{a(t_i)} - 1\right) - \frac{\epsilon}{2w^2}\frac{a(t_f)}{a(t_i)}\Big(a(t_f)^2 h(t_f, p_t^0(t_f, w), w)\\ &\qquad - a(t_i)^2 h(t_i, p_t^0(t_i, w), w)\Big)\,. \end{aligned} \tag{8}$$

To zeroth order, as expected, the redshift formula from general relativity is recovered, while the first order is determined by the perturbation h. In particular the perturbation depends in general on the particles spatial coordinate momentum w, which can be expressed in terms of the initial coordinate time-momentum of the photon $p_t(t_i)$, since equation (7) can be inverted for $w(p_t, t)$. Thus photons starting with different initial coordinate time-momentum $p_t^0(t_i, w)$ experience a different redshift. Hence a detection of a photon redshift dependent on the initial coordinate time-momentum is a clear signal for a modification of the dispersion relation while its absence puts constraints on the perturbation. First analyses of possible evidences for an energy dependent redshift have been performed [28,29].

We use the term coordinate time-momentum of a photon here instead of energy of a photon to distinguish between the observer dependent notion of energy of a particle and the observer independent choice of coordinates to describe the particle's four momentum.

3.2. Lateshift

To derive the lateshift we use again the Hamilton equations of motion (4), to solve for r parametrized in terms of the coordinate time

$$\begin{aligned} \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\dot r}{\dot t} &= \frac{\chi w}{a\sqrt{a^2 m^2 + w^2}}\Bigg(1 - \epsilon\frac{1}{2(p_t^0)^2}\Big[h(t, p_t^0, w)\\ &\qquad - p_t^0\partial_{p_t} h(t, p_t^0, w) - w\partial_w h(t, p_t^0, w)\Big]\Bigg)\\ &\equiv \frac{\chi w}{a\sqrt{a^2 m^2 + w^2}}(1 - \epsilon f(t, p_t^0, w))\,. \end{aligned} \tag{9}$$

The momentum corresponding to the time coordinate is considered as function $p_t = p_t(t, w, m)$ as displayed in (6). Employing separation of variables and the perturbative ansatz $r = r^0 + \epsilon r^1$ the following solution can easily be found