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The variable flavor number scheme at next-to-leading order

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ABSTRACT

We present the matching relations of the variable flavor number scheme at next-to-leading order, which are of importance to define heavy quark partonic distributions for the use at high energy colliders such as Tevatron and the LHC. The consideration of the two-mass effects due to both charm and bottom quarks, having rather similar masses, are important. These effects have not been considered in previous investigations. Numerical results are presented for a wide range of scales. We also present the corresponding contributions to the structure function $F_2(x, Q^2)$.

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In the variable flavor number scheme (VFNS), matching conditions are considered between parton distribution functions (PDFs) at N_F massless flavors and those at $N_F + k$ flavors (with usually $k = 1, 2$), at high factorization and renormalization scales μ^2. This allows to introduce heavy quark parton distribution functions, which are related to the quark-non-singlet, quark-singlet (Σ) and gluon (G) distributions via the universal massive operator matrix elements (OMEs) $A_{ij}^{(k)}(\mu^2, m_c^2, m_b^2)$. Likewise, the flavor non-singlet, singlet and gluon distribution functions receive corresponding QCD-corrections. In this paper we will work in the $\overline{\text{MS}}$-scheme in QCD, defining the heavy quark masses first in the on-shell scheme and later also transforming to the $\overline{\text{MS}}$-scheme. The VFNS for $k = 1$ has been discussed in Ref. [1] at NLO and at NNLO in [2] and including the two-mass effects in Ref. [3] to NNLO.

In the past the usual approach has been to deal with a single heavy quark at a time. This way has a longer tradition and is implemented in various applications. As lined out in [1] it assumes that the decoupling quark is massless at the next heavy quark threshold, e.g. charm can be considered massless at the bottom quark scale, etc. However, the charm and bottom quarks have rather similar masses with $m_c^2/m_b^2 \sim 1/10$ for their pole or $\overline{\text{MS}}$ masses at NLO and NNLO, which makes it difficult to assume $m_c^2 \ll m_b^2$, i.e. to consider the charm mass at $\mu = m_b$ massless. On the other hand, it is perfectly possible to decouple both quarks simultaneously and consider their effect at high scales $\mu \gg m_c, m_b$. In this note, we will describe the corresponding extension of the VFNS in this more general case at next-to-leading order.

The parton distributions for $N_F + 2$ flavors are related to those at N_F flavors by the following relations for the *number* densities in Mellin-N space, according to the renormalization prescription derived in Ref. [3], are:

$$f_{\mathrm{NS},i}(N_F+2,\mu^2) = \left\{1 + a_s^2(\mu^2)\left[A_{qq,Q}^{\mathrm{NS},(2,c)} + A_{qq,Q}^{\mathrm{NS},(2,b)}\right]\right\} f_{\mathrm{NS},i}(N_F,\mu^2), \quad (1)$$

$$\begin{aligned}\Sigma(N_F+2,\mu^2) &= \left\{1 + a_s^2(\mu^2)\left[A_{qq,Q}^{\mathrm{NS},(2,c)} + A_{qq,Q}^{\mathrm{PS},(2,c)} + A_{qq,Q}^{\mathrm{NS},(2,b)} + A_{qq,Q}^{\mathrm{PS},(2,b)}\right]\right\} \\ &\quad \times \Sigma(N_F,\mu^2) \\ &\quad + \left\{a_s(\mu^2)\left[A_{Qg}^{(1,c)} + A_{Qg}^{(1,b)}\right]\right. \\ &\quad \left. + a_s^2(\mu^2)\left[A_{Qg}^{(2,c)} + A_{Qg}^{(2,b)} + A_{Qg}^{(2,cb)}\right]\right\} G(N_F,\mu^2), \quad (2)\end{aligned}$$

$$G(N_F+2,\mu^2) = \left\{1 + a_s(\mu^2)\left[A_{gg,Q}^{(1,c)} + A_{gg,Q}^{(1,b)}\right]\right.$$

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$$+a_s^2(\mu^2)\Big[A_{gg,Q}^{(2,c)}+A_{gg,Q}^{(2,b)}+A_{gg,Q}^{(2,cb)}\Big]\Big\}G(N_F,\mu^2)$$
$$+a_s^2(\mu^2)\Big[A_{gq,Q}^{(2,c)}+A_{gq,Q}^{(2,b)}\Big]\Sigma(N_F,\mu^2), \qquad (3)$$

$$\Big[f_c+f_{\bar c}\Big](N_F+2,\mu^2)$$
$$=a_s^2(\mu^2)A_{Qq}^{\mathrm{PS},(2,c)}\Sigma(N_F,\mu^2)$$
$$+\left\{a_s(\mu^2)A_{Qg}^{(1,c)}+a_s^2(\mu^2)\Big[A_{Qg}^{(2,c)}+\frac{1}{2}A_{Qg}^{(2,cb)}\Big]\right\}G(N_F,\mu^2), \qquad (4)$$

$$\Big[f_b+f_{\bar b}\Big](N_F+2,\mu^2)$$
$$=a_s^2(\mu^2)A_{Qq}^{\mathrm{PS},(2,b)}\Sigma(N_F,\mu^2)$$
$$+\left\{a_s(\mu^2)A_{Qg}^{(1,b)}+a_s^2(\mu^2)\Big[A_{Qg}^{(2,b)}+\frac{1}{2}A_{Qg}^{(2,cb)}\Big]\right\}G(N_F,\mu^2)\,. \qquad (5)$$

Here $a_s=\alpha_s/(4\pi)=g_s^2/(4\pi)^2$ denotes the strong coupling constant in the $\overline{\mathrm{MS}}$ scheme. One easily sees in Eqs. (1)–(5) that three contributions are present in the transition from $N_F\to N_F+2$: (*i*) the pure charm contribution (c), (*ii*) the pure bottom contribution (b), allowing to reconstruct the single heavy flavor cases, and (*iii*) the combined charm and bottom contribution cb. While the former ones have been dealt with successively going from $N_F\to N_F+1\to N_F+2$, the mixed contributions have not been considered in Ref. [1], Eqs. (2.37–2.41), despite these terms are being implied by corresponding Feynman diagrams contributing to the respective OMEs. This decomposition holds as well in higher order, where also non-factorizing terms contribute. We will outline below the numerical effects on the different distributions in the transition $N_F\to N_F+2$.

The quark non-singlet and singlet distributions are defined by[1]

$$f_{\mathrm{NS},i}(N_F,\mu^2)=q_i(\mu^2)+\bar q_i(\mu^2), \qquad (6)$$
$$\Sigma(N_F,\mu^2)=\sum_{i=1}^{N_F}\Big[q_i(\mu^2)+\bar q_i(\mu^2)\Big]. \qquad (7)$$

The OMEs $A_{ij}^{(k,Q)}$ and $A_{ij}^{(k,Q_1Q_2)}$ depend on μ^2/m_Q^2 and $\mu^2/m_{Q_i}^2$ logarithmically. Eqs. (1)–(5) describe the corresponding heavy flavor contributions at N_F+2 flavors in fixed order perturbation theory. Here we have dropped the dependence on N of the contributing functions.

In x-space the convolutions are given by

$$([f]_+\otimes g)(x)=\int_x^1 dz\frac{1-z}{z}f(z)g\left(\frac{x}{z}\right)-g(x)\int_0^x dzf(z) \qquad (8)$$
$$(h\otimes g)(x)=\int_x^1\frac{dz}{z}h(z)g\left(\frac{x}{z}\right),\quad (\delta(1-x)\otimes g)(x)=g(x), \qquad (9)$$

for a +-distribution $[f]_+$, regular functions g and h, and the convolution with the $\delta(1-x)$-distribution. Again, we consider the case of number densities for $g(x)$ here. The +-distribution has the Mellin transform

$$\mathbf{M}[[f(x)]_+](N)=\int_0^1 dx\left[x^{N-1}-1\right]f(x)\,, \qquad (10)$$

and the Mellin transform in general obeys

$$\mathbf{M}[(h\otimes g)(x)](N)=\mathbf{M}[h(x)](N)\cdot\mathbf{M}[g(x)](N)\,. \qquad (11)$$

The flavor non-singlet distributions are not effected by two-mass terms at NLO, but first at NNLO, cf. [3,5]. The OMEs to NLO in Eqs. (1)–(5) have been calculated in Refs. [1,4,6–8] in the equal mass case. At NNLO the OMEs have been computed for a series of moments in [2] and for a part of the OMEs for general moments N in [5,8–15] in the equal mass case. In the unequal mass case at NNLO the moments $N=2,4,6$ of all OMEs were calculated in terms of an expansion in the mass ratio in [3] and a part of the general x corrections have been computed in [3,16,17] already.

The unequal mass corrections at NLO in Eqs. (1)–(5) were calculated in Ref. [3]. They are given by

$$A_{Qg}^{(2,cb)}=-\beta_{0,Q}\hat\gamma_{qg}^{(0)}\ln\left(\frac{\mu^2}{m_c^2}\right)\ln\left(\frac{\mu^2}{m_b^2}\right), \qquad (12)$$
$$A_{gg,Q}^{(2,cb)}=2\beta_{0,Q}^2\ln\left(\frac{\mu^2}{m_c^2}\right)\ln\left(\frac{\mu^2}{m_b^2}\right), \qquad (13)$$

where $\beta_{0,Q}=-(4/3)T_F$ and $T_F=1/2$ and

$$\hat\gamma_{qg}^{(0)}=-8T_F\frac{N^2+N+2}{N(N+1)(N+2)}, \qquad (14)$$

denotes the leading order splitting function for the process $g\to q$.[2] The following sum rule has to be obeyed due to energy-momentum conservation, cf. [2],

$$A_{Qg}(N=2)+A_{qg,Q}(N=2)+A_{gg,Q}(N=2)=1. \qquad (15)$$

The OME $A_{qg,Q}$ contributes from 3-loop order onwards only and has two heavy quark contributions only beginning at 4-loop order. The equal mass terms are already known to obey Eq. (15) up to $O(a_s^3)$, [2]. The NLO mass contributions add up to zero for $N=2$ in accordance with Eq. (15).

To illustrate the numerical effect of the NLO 2-mass terms on these distributions we consider the ratio

$$\alpha\frac{a_s^2(\mu^2)A_{ig}^{(2,cb)}G(N_F,\mu^2)}{\Phi(N_F+2,\mu^2)}, \qquad (16)$$

for $\Phi=\Sigma,G,(\alpha=1);[f_c+f_{\bar c}],[f_b+f_{\bar b}],(\alpha=1/2)$. In the case of the heavy flavor distributions, the effect is largest because it is of $O(a_s)$. A first simple estimate yields

$$\frac{\Big[f_c+f_{\bar c}\Big]^{\text{two mass}}(N_F+2,\mu^2)}{\Big[f_c+f_{\bar c}\Big]^{\text{all}}(N_F+2,\mu^2)}\approx a_s\left[\beta_{0,Q}\ln\left(\frac{\mu^2}{m_b^2}\right)+O(a_s)\right], \qquad (17)$$

and similar for $[f_b+f_{\bar b}]$ by exchanging $c\leftrightarrow b$. Here the leading term does not depend on the parton distributions in Mellin space.

For all contributions to the OMEs but $A_{Qg}^{(2)}$ and $A_{gg,Q}^{(2)}$ the same relation is obtained in the $\overline{\mathrm{MS}}$ and on-shell scheme to $O(a_s^2)$ for

[1] Actually, one should subtract from (6) the term Σ/N_F. However, as the functional relations are the same, we follow the convention suggested in Ref. [4].

[2] As very well known, splitting functions, to all orders in the coupling constant, are universal and do especially not contain power corrections m^2/Q^2 stemming e.g. from phase space corrections.

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