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A remark on the sign change of the four-particle azimuthal cumulant in small systems

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ABSTRACT

The azimuthal cumulants, $c_2\{2\}$ and $c_2\{4\}$, originating from the global conservation of transverse momentum in the presence of hydro-like elliptic flow are calculated. We observe the sign change of $c_2\{4\}$ for small number of produced particles. This is in a qualitative agreement with the recent ATLAS measurement of multi-particle azimuthal correlations with the subevent cumulant method.

1. Introduction

Experimental results from heavy-ion colliders indicate that a nearly perfect fluid is produced in high energy nucleus–nucleus (A + A) collisions [1–3]. One important evidence is the success of hydrodynamics in describing the collective flow phenomena observed in A + A, see, e.g., [4–9]. The hydrodynamical models capture the main features of collective flow measured using different methods [10–14]. For example, the k-particle azimuthal cumulants, $c_n\{k\}$, are expected to measure the *real* collective flow v_n by reducing non-flow effects [11,12]. The experimental results from the Large Hadron Collider (LHC) show that the elliptic flow coefficients obtained with four, six and eight-particle standard cumulant method are overlapping in both Pb + Pb and p + Pb collisions, indicating that the observed long-range (in rapidity) azimuthal correlations may be due to the same physical origin in both large and small systems [15–17].

A new subevent cumulant method was recently developed to further suppress the non-flow contribution from jets [18]. The ATLAS measurement [19] demonstrated that the two-subevent and three-subevent cumulants are less sensitive to short-range non-flow effects than the standard cumulant method. The three-subevent method shows that $c_2\{4\}$ in proton–proton and p + Pb collisions changes sign at lower multiplicity than the standard method, indicating that the long-range multi-particle azimuthal correlations persist to even lower multiplicities. On the other hand, many theoretical efforts have been made to understand these measurements, which are basically classified as final state [20–30] or initial state phenomena [31–40], see [41] for a recent review.

In this paper we calculate the two-particle and the four-particle azimuthal cumulants

$$c_2\{2\} = \left\langle e^{i2(\phi_1-\phi_2)} \right\rangle, \tag{1}$$

$$c_2\{4\} = \left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle^2, \tag{2}$$

originating from the conservation of transverse momentum in the presence of hydro-like elliptic flow.

Recently we calculated the effect of transverse momentum conservation (TMC) only [42], and we observed that

$$c_2\{k\} \sim \frac{1}{N^k}, \tag{3}$$

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with $c_2\{k\} > 0$ for the calculated $k = 2, 4, 6, 8$.[1] Here N is the number of produced particles subjected to TMC. As shown in [42], the contribution from TMC to $(c_2\{k\})^{1/k}$ is of the order of a few percent even for a relatively large number of particles. In this paper we extend our analysis and calculate analytically $c_2\{2\}$ and $c_2\{4\}$ originating from TMC applied to particles characterized by the hydro-like elliptic flow. We observe that $c_2\{4\}$ changes sign for small N in a qualitative agreement with the recent ATLAS measurement of multi-particle azimuthal correlations with the subevent cumulant method [18,19].

2. Calculation

We calculate the effect of TMC applied to particles characterized by the hydro-like elliptic flow. This can be modeled by a single particle distribution given by[2]

$$f(p,\phi) = \frac{g(p)}{2\pi}\left[1 + 2v_2(p)\cos(2\phi - 2\Psi_2)\right], \tag{4}$$

where $v_2(p)$ is the elliptic flow at a given transverse momentum $p = |\vec{p}|$. Ψ_2 is the event plane, which we further put to zero.

2.1. Two particles

Following calculations presented, e.g., in Refs. [42–48], the two-particle distribution with TMC is given by

$$f_2(p_1,\phi_1,p_2,\phi_2) = f(p_1,\phi_1) f(p_2,\phi_2)\frac{N}{N-2}\exp\left(-\frac{(p_{1,x}+p_{2,x})^2}{2(N-2)\left\langle p_x^2\right\rangle_F} - \frac{(p_{1,y}+p_{2,y})^2}{2(N-2)\left\langle p_y^2\right\rangle_F}\right), \tag{5}$$

where $p_x = p\cos(\phi)$, $p_y = p\sin(\phi)$ and using Eq. (4) we have

$$\begin{aligned}\left\langle p_x^2\right\rangle_F &= \frac{1}{2}\left\langle p^2\right\rangle_F\left(1+\bar{\bar{v}}_{2,F}\right),\\ \left\langle p_y^2\right\rangle_F &= \frac{1}{2}\left\langle p^2\right\rangle_F\left(1-\bar{\bar{v}}_{2,F}\right),\end{aligned} \tag{6}$$

where

$$\bar{\bar{v}}_{2,F} = \frac{\left\langle v_2(p)p^2\right\rangle_F}{\left\langle p^2\right\rangle_F} = \frac{\int_F g(p)v_2(p)p^2d^2p}{\int_F g(p)p^2d^2p}. \tag{7}$$

The integrations over the full phase space are always denoted by F.

Our goal is to calculate

$$\langle e^{2i(\phi_1-\phi_2)}\rangle|_{p_1,p_2} = \frac{\int_0^{2\pi} f_2(p_1,\phi_1;p_2,\phi_2)e^{2i(\phi_1-\phi_2)}d\phi_1 d\phi_2}{\int_0^{2\pi} f_2(p_1,\phi_1;p_2,\phi_2)d\phi_1 d\phi_2} = \frac{U_2}{D_2}, \tag{8}$$

where $\langle e^{2i(\phi_1-\phi_2)}\rangle$ is calculated at a given transverse momenta p_1 and p_2.

To calculate the numerator we expand $\exp(-A) \approx 1 - A + A^2/2$ and neglect all higher terms in Eq. (5). As shown in Ref. [42] the first contribution from TMC, which is not vanishing at $v_2 = 0$, appears in $A^2/2$. We obtain[3]

$$\begin{aligned}\frac{U_2}{4\pi^2} = {} & v_2(p_1)v_2(p_2) - \frac{p_1^2 v_2(p_2)[2v_2(p_1)-\bar{\bar{v}}_{2,F}] + p_2^2 v_2(p_1)[2v_2(p_2)-\bar{\bar{v}}_{2,F}]}{2(N-2)\left\langle p^2\right\rangle_F[1-(\bar{\bar{v}}_{2,F})^2]} + \\ & \frac{p_1^4 v_2(p_2)[v_2(p_1)\{4+3(\bar{\bar{v}}_{2,F})^2\}-4\bar{\bar{v}}_{2,F}] + p_2^4 v_2(p_1)[v_2(p_2)\{4+3(\bar{\bar{v}}_{2,F})^2\}-4\bar{\bar{v}}_{2,F}]}{8(N-2)^2\left\langle p^2\right\rangle_F^2[1-(\bar{\bar{v}}_{2,F})^2]^2} + \\ & \frac{2p_1^2p_2^2[4v_2(p_1)v_2(p_2)\{2+(\bar{\bar{v}}_{2,F})^2\}-6\bar{\bar{v}}_{2,F}\{v_2(p_1)+v_2(p_2)\}+(\bar{\bar{v}}_{2,F})^2]}{8(N-2)^2\left\langle p^2\right\rangle_F^2[1-(\bar{\bar{v}}_{2,F})^2]^2} + \frac{p_1^2p_2^2}{2(N-2)^2\left\langle p^2\right\rangle_F^2[1-(\bar{\bar{v}}_{2,F})^2]^2}.\end{aligned} \tag{9}$$

To calculate the denominator it is enough to take the first term, $\exp(-A) \approx 1$, since the next terms are suppressed by the powers of $1/N$. In this case we obtain

$$D_2 = 4\pi^2, \tag{10}$$

and the first correction (assuming $v_2^2 \ll 1$) is given by $-4\pi^2\frac{p_1^2+p_2^2}{(N-2)\langle p^2\rangle_F}$.

The last term of U_2 in Eq. (9), discussed in Ref. [42], is driven by momentum conservation and it does not vanish for $v_2 = 0$. It scales like $1/N^2$. The third and the fourth terms of U are suppressed also by $1/N^2$ and additionally they are multiplied by v_2^2, and

[1] For comparison, clusters decaying into k particles result in $c_2\{k\} \sim 1/N^{k-1}$, see, e.g., Ref. [11].

[2] We neglect v_3 which also contributes to $c_2\{2\}$ and $c_2\{4\}$ however, its effect is smaller than v_2.

[3] We skip $\frac{g(p_1)}{2\pi}\frac{g(p_2)}{2\pi}\frac{N}{N-2}$ appearing in Eq. (5) since it cancels in the ratio U_2/D_2.