Physics Letters B 782 (2018) 528–532

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Heavy quark form factors at three loops in the planar limit

J. Ablinger [a], J. Blümlein [b], P. Marquard [b,*], N. Rana [b], C. Schneider [a]

[a] Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Altenbergerstraße 69, A-4040, Linz, Austria
[b] Deutsches Elektronen–Synchrotron, DESY, Platanenallee 6, D-15738 Zeuthen, Germany

ARTICLE INFO

Article history:
Received 25 April 2018
Accepted 29 May 2018
Available online 31 May 2018
Editor: A. Ringwald

ABSTRACT

We compute the color-planar and complete light quark non-singlet contributions to the heavy quark form factors in the case of the axialvector, scalar and pseudoscalar currents at three loops in perturbative QCD. We evaluate the master integrals applying a new method based on differential equations for general bases, which is applicable for all first order factorizing systems. The analytic results are expressed in terms of harmonic polylogarithms and real-valued cyclotomic harmonic polylogarithms.

Form factors are the matrix elements of local composite operators between physical states. In perturbative Quantum Chromodynamics (QCD) these objects play a significant role in determining physical observables. In scattering cross-sections, they provide important contributions to the virtual corrections. The massive form factors are of importance for the forward–backward asymmetry of bottom or top quark pair production at electron–positron colliders and to static quantities like the anomalous magnetic moment of a heavy quark and other processes. They are also of importance to scrutinize the properties of the top quark [1,2] during the high luminosity phase of the LHC [3] and the experimental precision studies at future high energy e^+e^- colliders [4].

In this letter, we calculate both the color-planar and complete light quark non-singlet three-loop contributions to the massive form factors for axialvector, scalar and pseudoscalar currents. Our results for the vector current, including a detailed account of the techniques used in these calculations, will be presented elsewhere [5]. The two-loop QCD corrections to the massive vector, axialvector form factors, the anomaly contributions, and the scalar and pseudoscalar form factors were first presented in [6–9]. In [10], an independent computation led to a cross-check of the vector form factor, giving also the additional $\mathcal{O}(\varepsilon)$ terms in the dimensional parameter $\varepsilon = (4-D)/2$. Recently, the contributions up to $\mathcal{O}(\varepsilon^2)$ for all the massive two-loop form factors were obtained in Ref. [11]. The color-planar contributions to the massive three-loop form factor for the vector current have been computed in [12,13] and the complete light quark contributions in [14]. The large β_0 limit has been considered in [15].

Our notations follow those used in Ref. [11]. We consider the decay of a virtual massive boson of momentum q into a pair of heavy quarks of mass m, momenta q_1 and q_2 and color c and d, through a vertex X_{cd}, where $X_{cd} = \Gamma^{\mu}_{A,cd}, \Gamma_{S,cd}$ and $\Gamma_{P,cd}$ indicates the coupling to an axialvector, a scalar and a pseudoscalar boson, respectively. Here $q^2 = (q_1+q_2)^2$ is the center of mass energy squared and the dimensionless variable s is defined by

$$s = \frac{q^2}{m^2}\,. \tag{1}$$

The amplitudes take the following general form

$$\bar{u}_c(q_1) X_{cd} v_d(q_2)\,, \tag{2}$$

where $\bar{u}_c(q_1)$ and $v_d(q_2)$ are the bi-spinors of the quark and the anti-quark, respectively. We denote the corresponding UV renormalized form factors by F_I, with $I = A, S, P$. They are expanded in the strong coupling constant $\alpha_s = g_s^2/(4\pi)$ as follows

$$F_I = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n F_I^{(n)}\,. \tag{3}$$

The following generic forms for the amplitudes can be found by studying the general Lorentz structure. For the axialvector current, it can be cast as

$$\Gamma^{\mu}_{A,cd} = -i\delta_{cd}\Big[a_Q\left(\gamma^{\mu}\gamma_5\, F_{A,1} + \frac{1}{2m} q^{\mu}\gamma_5\, F_{A,2}\right)\Big], \tag{4}$$

where a_Q is the Standard Model (SM) axialvector coupling constant. Likewise, for the scalar and pseudoscalar currents, one has

$$\Gamma_{cd} = \Gamma_{S,cd} + \Gamma_{P,cd} = -\frac{m}{v}\delta_{cd}\left[s_Q\, F_S + i p_Q\, \gamma_5\, F_P\right], \tag{5}$$

* Corresponding author.
E-mail address: peter.marquard@desy.de (P. Marquard).

https://doi.org/10.1016/j.physletb.2018.05.077

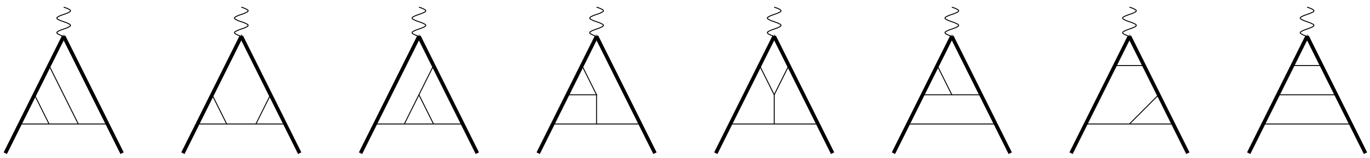

Fig. 1. The color-planar topologies.

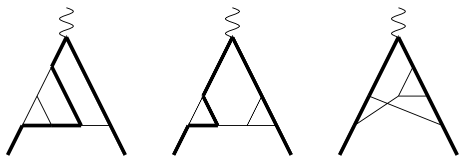

Fig. 2. The n_l topologies.

where $v = (\sqrt{2}G_F)^{-1/2}$ is the SM vacuum expectation value of the Higgs field, with G_F being the Fermi constant, s_Q and p_Q are the scalar and pseudoscalar couplings, respectively. Finally, to obtain the unrenormalized form factors, we multiply appropriate projectors as provided in [11] and perform the trace over the color and spinor indices. For later purpose we denote by N_c the number of colors, and n_l and n_h are the number of light and heavy quarks, respectively. We will set $n_h = 1$ in the following.

Since we use dimensional regularization [16], one important point is to define a proper description for the treatment of γ_5. Both the color-planar and complete n_l contribution belong to the so-called non-singlet case, where the axialvector or pseudoscalar vertex is connected to open heavy fermion lines. Hence, both γ_5-matrices appear in the same chain of Dirac matrices, which allows us to use an anti-commuting γ_5 in D space–time dimensions, with $\gamma_5^2 = 1$. This is implied by the well-known Ward identity,

$$q^\mu \Gamma^{\mu,\mathrm{ns}}_{A,cd} = 2m\Gamma^{\mathrm{ns}}_{P,cd}\,, \tag{6}$$

which in terms of the form factors, takes the form

$$2F^{\mathrm{ns}}_{A,1} + \frac{s}{2}F^{\mathrm{ns}}_{A,2} = 2F^{\mathrm{ns}}_{P}\,. \tag{7}$$

Here ns denotes the non-singlet contributions. For convenience, we introduce the kinematic variable [17]

$$x = \frac{\sqrt{q^2-4m^2}-\sqrt{q^2}}{\sqrt{q^2-4m^2}+\sqrt{q^2}} \quad \leftrightarrow \quad s = \frac{q^2}{m^2} = -\frac{(1-x)^2}{x}, \tag{8}$$

which we use in the following. In particular, we focus on the Euclidean region, $q^2 < 0$, corresponding to $x \in [0, 1[$.

The Feynman diagrams for the different form factors are generated using QGRAF [18], the color algebra is performed using Color [19], the output of which is then processed using Q2e/Exp [20,21] and FORM [22,23] in order to express the diagrams in terms of a linear combination of a large set of scalar integrals. These integrals are then reduced using integration by parts identities (IBPs) [24,25] with the help of the program Crusher [26] to obtain 109 master integrals (MIs), out of which 96 appear in the color-planar case. In the color-planar limit, the families of integrals can be represented by eight topologies, shown in Fig. 1, whereas for the complete light quark contributions, three more topologies, cf. Fig. 2, are required.[1]

[1] Only sub-topologies with a maximum of eight propagators contribute.

Finally, the master integrals have to be computed. For this we use the method of differential equations, see also [27–30]. The corresponding differential equations are obtained from the IBP relations. Here a central question is whether the corresponding linear system of differential equations is first order factorizable or not. Using the package Oresys [31], based on Zürcher's algorithm [32,33], we have proved that the present system is indeed first order factorizable in x-space. Without any need to choose a special basis, one is therefore in the position to solve the system in terms of iterated integrals of whatsoever alphabet, cf. Ref. [5] for details. The differential equations are solved order by order in ε successively, starting at the leading pole terms $\propto 1/\varepsilon^3$. The successive solutions in ε contribute to the inhomogeneities in the next order. We compute the master integrals block-by-block, where for an $m \times m$ system a single inhomogeneous ordinary differential equation of order m or less is obtained, which we solved using the variation of constant. The other $m-1$ solutions result from the former solution immediately. The boundary conditions can be determined by a separate calculation at $x = 1$. The calculation is performed by intense use of HarmonicSums [34–40], which uses the package Sigma [41,42]. We finally have checked all master integrals numerically using FIESTA [43–45].

In the present case, the emerging harmonic polylogarithms stem from the inhomogeneities, adding further letters which result from the rational coefficients in the differential equations. They are obtained by partial fractioning as the k-th powers of letters, $k \in \mathbb{N}$, which have to be transformed to the letters by partial integration in case. This method has some relation to the method of hyperlogarithms [46,47]. One obtains up to weight $\mathrm{w} = 6$ real-valued iterated integrals over the alphabet

$$\frac{1}{x},\ \frac{1}{1-x},\ \frac{1}{1+x},\ \frac{1}{1-x+x^2},\ \frac{x}{1-x+x^2}, \tag{9}$$

i.e. the usual harmonic polylogarithms (HPLs) [48] and their cyclotomic extension [34], including the respective constants in the limit $x \to 1$, i.e. the multiple zeta values (MZVs) [49] and the cyclotomic constants [34,50]. In case of the iterated integrals we apply the linear representation. For a numerical implementation the use of the shuffle algebra [51] implemented in HarmonicSums reduces the number of functions accordingly. In the MZV and cyclotomic case there are proven reduction relations to weight $\mathrm{w} = 12$ [49] and $\mathrm{w} = 5$ [50], respectively, which we have used. The 64 cyclotomic constants which appear up to $\mathrm{w} = 5$ reduce to 18. At $\mathrm{w} = 6$ 124 cyclotomic constants remain at the moment. Note that there are more conjectured relations, cf. [52], based on PSLQ [53]. If these conjectured relations are used, only multiple zeta values remain as constants in all form factors using our real representation for the cyclotomic harmonic polylogarithms. The analytic result for the different form factors in terms of HPLs and cyclotomic HPLs [34,48] can be analytically continued outside $x \in [0, 1[$ by using the mappings $x \to -x, x \to (1-x)/(1+x)$ on the expense of extending the cyclotomy class in cases needed.

The UV renormalization of the form factors has been performed in a mixed scheme. We renormalize the heavy quark mass and