Physics Letters B 781 (2018) 161–168

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Matching factorization theorems with an inverse-error weighting

Miguel G. Echevarria [a,*], Tomas Kasemets [b], Jean-Philippe Lansberg [c], Cristian Pisano [d], Andrea Signori [e]

[a] INFN, Sezione di Pavia, Via Bassi 6, 27100 Pavia, Italy
[b] PRISMA Cluster of Excellence & Mainz Institute for Theoretical Physics, Johannes Gutenberg University, 55099 Mainz, Germany
[c] IPNO, CNRS-IN2P3, Univ. Paris-Sud, Université Paris-Saclay, 91406 Orsay Cedex, France
[d] Dipartimento di Fisica, Università di Cagliari and INFN, Sezione di Cagliari, Cittadella Universitaria, I-09042 Monserrato (CA), Italy
[e] Theory Center, Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA

ARTICLE INFO

Article history:
Received 12 January 2018
Received in revised form 27 March 2018
Accepted 28 March 2018
Available online 3 April 2018
Editor: A. Ringwald

ABSTRACT

We propose a new fast method to match factorization theorems applicable in different kinematical regions, such as the transverse-momentum-dependent and the collinear factorization theorems in Quantum Chromodynamics. At variance with well-known approaches relying on their simple addition and subsequent subtraction of double-counted contributions, ours simply builds on their weighting using the theory uncertainties deduced from the factorization theorems themselves. This allows us to estimate the unknown complete matched cross section from an inverse-error-weighted average. The method is simple and provides an evaluation of the theoretical uncertainty of the matched cross section associated with the uncertainties from the power corrections to the factorization theorems (additional uncertainties, such as the nonperturbative ones, should be added for a proper comparison with experimental data). Its usage is illustrated with several basic examples, such as Z boson, W boson, H^0 boson and Drell–Yan lepton-pair production in hadronic collisions, and compared to the state-of-the-art Collins–Soper–Sterman subtraction scheme. It is also not limited to the transverse-momentum spectrum, and can straightforwardly be extended to match any (un)polarized cross section differential in other variables, including multi-differential measurements.

1. Motivation

In processes with a hard scale Q and a measured transverse momentum q_T, for instance the mass and the transverse momentum of an electroweak boson produced in proton–proton collisions, the q_T-differential cross section can be expressed through two different factorization theorems. For small $q_T \ll Q$, the transverse-momentum-dependent (TMD) factorization applies and the cross section is factorized in terms of TMD parton distribution/fragmentation functions (TMDs thereafter) [1–3]. The evolution of the TMDs resums the large logarithms of Q/q_T [4–6]. For large $q_T \sim Q \gg m$, with m a hadronic mass of the order of 1 GeV, there is only one hard scale in the process and the collinear factorization is the appropriate framework. The cross section is then written in terms of (collinear) parton distribution/fragmentation functions (PDFs/FFs). In order to describe the full q_T spectrum, the TMD and collinear factorization theorems must properly be matched in the intermediate region.

Many recent works on TMD phenomenology and extractions of TMDs from data did not take into account the matching with fixed-order collinear calculations for increasing transverse momentum (see e.g. Refs. [7,8]). Such a matching is one of the compelling milestones for the next generation of TMD analyses and more generally for a thorough understanding of TMD observables [9]. In addition, it has recently been shown that the precisely measured transverse-momentum spectrum of Z boson at the LHC does not completely agree with collinear-based NNLO computations,[1] hinting at possible higher-twist contributions at the per-cent level. Thus having a reliable estimation of the matching uncertainty from power corrections is very opportune.

* Corresponding author.
E-mail address: mgechevarria@pv.infn.it (M.G. Echevarria).

[1] See https://gsalam.web.cern.ch/gsalam/talks/repo/2016-03-SB+SLAC-SLAC-precision.pdf.

https://doi.org/10.1016/j.physletb.2018.03.075

This work contributes to this effort by introducing a new approach, whose main features are its simplicity and its easy and fast implementation in phenomenological analyses (fits and/or Monte Carlo event generators). In addition, this scheme provides an automatic estimate of the theoretical uncertainty associated to the matching procedure. All these are crucial features in light of the computational demands of global TMD analyses and event generation for the next generation of experiments [10–13].

As we will show, it yields compatible results with other mainstream approaches in the literature, such as the improved Collins–Soper–Sterman (CSS) scheme [14] (see also Ref. [15]), which refines the original CSS subtraction approach [16–19]. The latter, in simple terms, is based on adding the TMD-based resummed ($\mathcal{W}$) and collinear-based fixed-order ($\mathcal{Z}$) results, and then subtracting the double-counted contributions ($\mathcal{A}$). The improved CSS (*iCSS*) approach enforces the necessary cancellations for the subtraction method to work.

Other methods have been introduced in the framework of soft-collinear effective theory by using profile functions for the resummation scales in order to obtain analogous cancellations to those in the *iCSS* method, see e.g. Refs. [20–23]. One can also find other schemes to match TMD and collinear frameworks, e.g. Refs. [24–26].

In the scheme we introduce, no cancellation between the TMD-based resummed contribution, $\mathcal{W}$, and the collinear-based fixed-order contribution, $\mathcal{Z}$, is needed. We simply avoid the double counting (and therewith the subtraction of $\mathcal{A}$) by weighting both contributions to the matched cross section, with the condition that the weights add up to unity. This renders the computation of the matched cross section very easy to implement. Clearly, the weights cannot be arbitrary and should ensure that, in their respective domains of applicability, the predictions of both factorization theorems are recovered.

Both factorized expressions can be seen as approximations of the unknown, *true* theory, up to corrections expressed as ratios of the relevant scales (power corrections, in the following). In TMD factorization the power corrections scale as a power of q_T/Q, whereas in collinear factorization they scale as a power of m/q_T, up to further suppressed nonperturbative contributions [1]. We simply implement an estimate of these uncertainties in the well-known formula of an inverse-error weighting – or inverse-variance weighted average – of two measurements to obtain our matched predictions. As such, it also automatically returns an evaluation of the corresponding matching uncertainty.

The method we propose can straightforwardly be extended to match any (un)polarized cross section differential in other variables, including for instance event shapes, multi-differential measurements or double parton scattering with a measured transverse momentum [27].

This paper is organized as follows: in Sec. 2 we describe both factorization theorems for low and high transverse momenta, and how they are combined with the inverse-error-weighting method. In Sec. 3 we show through several examples (Z, W, H^0 and Drell–Yan lepton-pair production) how the method works. In Sec. 4 we compare the numerical results to the *iCSS* subtraction scheme. Finally, Sec. 5 gathers the conclusions and briefly discusses the applicability of our method to other processes.

2. The inverse-error weighting method

The main idea behind the scheme we are proposing is to use the power corrections to the involved factorization theorems in order to directly determine to which extent the approximations can be trusted in different kinematic regions, and to use this in order to bridge the intermediate region obtaining the complete spectrum. In this context, an inverse-error weighting is conceptually the simplest method one could think of.

Let us have a closer look at the TMD and collinear factorization theorems and their regions of validity, by considering a cross section $d\sigma$ differential in at least the transverse momentum q_T of an observed particle. For $q_T \ll Q$, the TMD factorization can reliably be applied and the q_T-differential cross section can generically be written as

$$d\sigma(q_T, Q)\Big|_{q_T \ll Q} = \mathcal{W}(q_T, Q) + \left[\mathcal{O}\left(\frac{q_T}{Q}\right)^{a} + \mathcal{O}\left(\frac{m}{Q}\right)^{a'}\right] d\sigma(q_T, Q)\,, \tag{1}$$

where $\mathcal{W}$ is the TMD approximation of the cross section $d\sigma$, the scale m is a hadronic mass scale on the order of 1 GeV and Q is the hard scale in the process, for instance the invariant mass of the produced particle. As q_T increases, the accuracy of the TMD approximation decreases and the power corrections are increasingly relevant until the expansion breaks down as q_T approaches Q.

On the contrary, for large $q_T \sim Q \gg m$, the collinear factorization theorem applies and the q_T-differential cross section can generically be written as

$$d\sigma(q_T, Q)\Big|_{q_T \sim Q \gg m} = \mathcal{Z}(q_T, Q) + \mathcal{O}\left(\frac{m}{q_T}\right)^{b} d\sigma(q_T, Q)\,, \tag{2}$$

where $\mathcal{Z}$ is the collinear approximation of the full cross section $d\sigma$. $\mathcal{Z}$ is calculated at a fixed-order in the strong coupling constant α_s. For $q_T \sim Q \gg m$, $\mathcal{Z}$ is a good approximation of the full cross section, but as q_T decreases the accuracy of the collinear approximation diminishes, which finally breaks down as q_T approaches m.

Armed with both these factorization theorems, valid in different and (sometimes) overlapping regions, the full q_T spectrum can be constructed through a matching scheme. Such a scheme must make sure that the result agrees with $\mathcal{W}$ in the small q_T region and with $\mathcal{Z}$ in the large q_T region, and that there is a smooth transition in the intermediate region.

As announced, in this paper we introduce a new scheme, the *inverse-error weighting* (*InEW* for short), where the power corrections to the factorization theorems are used to quantify the trustworthiness associated to the respective contributions, and thus employed to build a weighted average. The resulting matched differential cross section over the full range in q_T is given by

$$\overline{d\sigma}(q_T, Q) = \omega_1 \mathcal{W}(q_T, Q) + \omega_2 \mathcal{Z}(q_T, Q)\,, \tag{3}$$

where the normalized weights for each of the two terms are

$$\omega_1 = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}\,, \qquad \omega_2 = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}\,, \tag{4}$$

with $\Delta\mathcal{W}$ and $\Delta\mathcal{Z}$ being the uncertainties of both factorization theorems generated by their power corrections. The uncertainty on the matched cross section simply follows from the propagation of these (uncorrelated) theory uncertainties:

$$\Delta\overline{d\sigma} = \frac{1}{\sqrt{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}} = \frac{\Delta_{\mathcal{W}}\Delta_{\mathcal{Z}}}{\sqrt{\Delta_{\mathcal{W}}^2 + \Delta_{\mathcal{Z}}^2}}\, d\sigma \approx \frac{\Delta_{\mathcal{W}}\Delta_{\mathcal{Z}}}{\sqrt{\Delta_{\mathcal{W}}^2 + \Delta_{\mathcal{Z}}^2}}\, \overline{d\sigma}\,, \tag{5}$$

where $\{\Delta\mathcal{W}, \Delta\mathcal{Z}\} = \{\Delta_{\mathcal{W}}, \Delta_{\mathcal{Z}}\} d\sigma$, and in the last step we have replaced the unknown true cross section $d\sigma$ by its estimated