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Tensor modes in pure natural inflation

Yasunori Nomura [a,b,c], Masahito Yamazaki [c,*]

[a] Berkeley Center for Theoretical Physics, Department of Physics, University of California, Berkeley, CA 94720, USA
[b] Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
[c] Kavli Institute for the Physics and Mathematics of the Universe (WPI), University of Tokyo, Kashiwa, Chiba 277-8583, Japan

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ABSTRACT

We study tensor modes in pure natural inflation [1], a recently-proposed inflationary model in which an axionic inflaton couples to pure Yang–Mills gauge fields. We find that the tensor-to-scalar ratio r is naturally bounded from below. This bound originates from the finiteness of the number of metastable branches of vacua in pure Yang–Mills theories. Details of the model can be probed by future cosmic microwave background experiments and improved lattice gauge theory calculations of the θ-angle dependence of the vacuum energy.

Cosmic inflation is a successful framework in explaining many distinguished features of our Universe, including its flatness and the origin of primordial density perturbations. There are, however, a plethora of inflationary models proposed in the literature, and we ultimately need to turn to observations for guidance, to convincingly answer the question of exactly which inflationary model describes our Universe.

Future detection of primordial tensor modes in comic microwave background (CMB) radiation would be ideal for this purpose. The size of primordial tensor modes is quantified by the tensor-to-scalar ratio r, and when combined with the observed value of the scalar spectral index n_s, these two parameters severely constrain models of inflation. This therefore provides an exciting opportunity for narrowing down possible models, especially because values of $r \sim 10^{-3}$ are expected to be within reach in next-generation CMB measurements (see e.g. Ref. [2]).

The goal of this paper is to study the prediction for tensor modes of the recently-proposed inflationary model of *pure natural inflation* [1]. This is arguably the simplest model of inflation consistent with the current observational data. It is defined within conventional low-energy effective field theory and is technically natural, i.e. stable under quantum corrections.

The model is given by an axionic inflaton ϕ coupling to four-dimensional pure Yang–Mills gauge fields:

$$\mathcal{L}_{\phi FF} = \frac{1}{32\pi^2}\frac{\phi}{f}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}, \tag{1}$$

where f is the decay constant and the dimensionless combination $\theta := \phi/f$ plays the role of the θ-angle of the Yang–Mills theory. Below we choose the Yang–Mills gauge group to be $SU(N)$ for simplicity.

The inflaton potential $V(\phi)$ is determined by the dynamics of the pure Yang–Mills theory. For our purposes, it is useful to parameterize the potential in the form

$$V(\phi) = M^4\left[1 - \frac{1}{(1+(\phi/F)^2)^p}\right]. \tag{2}$$

Here, M and F are two parameters which have dimensions of mass, and the exponent $p > 0$ is a dimensionless parameter. The parameter F plays the role of the effective decay constant.

This potential is motivated by the holographic computation of Ref. [3], which gives the parameters M and F to be

$$M \approx \sqrt{N}\Lambda, \qquad F \approx Nf, \tag{3}$$

where Λ is the dynamical scale of the Yang–Mills theory. We define the parameter γ by

$$F = \pi\gamma Nf. \tag{4}$$

As we will see later, $\gamma \approx O(1)$. For our purposes, we use γ and the power p to parameterize the strong-coupling dynamics of the Yang–Mills theory.[1]

* Corresponding author.
E-mail address: masahito.yamazaki@ipmu.jp (M. Yamazaki).

[1] The holographic result in Ref. [3] gives $p = 3$. We will not be restricted to this specific value; see Ref. [1].

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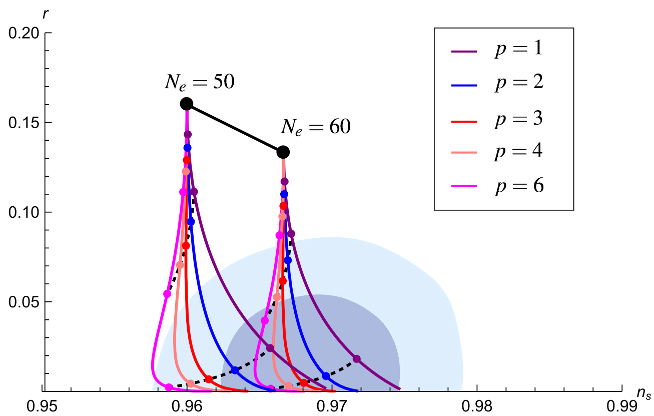

Fig. 1. The values of n_s and r predicted by the model for the number of e-folds $N_e = 50, 60$ and for $p = 1, 2, 3, 4, 6$. The light (dark) blue region represents the 95% (68%) CL allowed region by Planck [6] and BICEP2/Keck Array [7]. $F/M_{\rm Pl} = 10, 5, 1$ are indicated by dots (from top to bottom). This plot is the same as that in Fig. 3 of Ref. [1], except for the choice of the values of p. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

The potential of Eq. (2) takes approximately the quadratic form $V(\phi) \sim \phi^2$ for $\phi \ll F$, but it begins to deviate from this form as ϕ/F becomes larger. Note that this potential is rather different from the cosine potential used in the original model of natural inflation [4,5], which is motivated by the instanton approximation—as explained in Ref. [1], the cosine potential is not theoretically valid for pure Yang–Mills theory, and is also disfavored by the recent observations by Planck [6] and BICEP2/Keck Array [7].

The parameter M in Eq. (2) is determined by the overall size of the scalar perturbation once the other parameters, F and p, are given. While the power spectrum depends on all these parameters, in the range of F and p considered in this paper, we find

$$M \sim 10^{16}\ \text{GeV}. \tag{5}$$

This implies that to discuss the tensor-to-scalar ratio r and spectral index n_s, only the effective decay constant F and the power p are relevant. When we vary these parameters, we obtain a range of r and n_s which are in impressive agreement with the current observational constraints; see Fig. 1.

We see that the value of the spectral index n_s is mostly consistent with observation regardless of the values of F and p. On the other hand, the size of the tensor-to-scalar ratio r strongly depends on the value of F. Our main interest in this paper is to figure out the expected size of the tensor-to-scalar ratio r, or equivalently the value of F, in the present model.

If F is large, $F \gtrsim M_{\rm Pl}$, we expect to have a large value of r, and hence tensor modes can be observationally found in the near future. Here, $M_{\rm Pl} \simeq 1.22 \times 10^{19}$ GeV is the Planck scale. In the limit that F is very large, $F \gg M_{\rm Pl}$, the prediction of the model approaches that of chaotic inflation [8] with the quadratic potential $V(\phi) = m^2\phi^2/2$, which is now excluded at about a 3σ level [6,7]. However, as discussed in our previous paper [1], this extreme limit is not available in our framework, since the validity of low-energy effective field theory puts a constraint $F \lesssim O(M_{\rm Pl})$.

In the opposite limit of small F, the tensor-to-scalar ratio r is small; in fact, it can be tiny if F is much smaller than $M_{\rm Pl}$. At first sight, there seems to be no issue in going to this extreme parameter region. The predicted value of n_s is consistent with current experimental bounds, as can be seen in Fig. 1. The necessary amount of inflation, $N_e \approx 50$–60, can be obtained if the initial value of the inflaton field is large, $\phi \gg F$. However, there is a reason to think that such a parameter region may not be available in the model. This has to do with the fact that the potential in Eq. (2) is motivated by the holographic computation in the large N limit, and it should not be taken at face value once we taken into account the finite N effects.

To explain this point (in the language of quantum field theory), let us first recall the salient features of the large N analysis [9,10].

In addition to the axion coupling in Eq. (1), we have the kinetic term for the gauge fields, so that the total Lagrangian density is given by

$$\mathcal{L} = N\left[-\frac{1}{4\lambda}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{32\pi^2}\frac{\phi}{Nf}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}\right]. \tag{6}$$

Here, we have factored out the overall coefficient N, and $\lambda = g^2 N$ is the 't Hooft coupling with g being the gauge coupling. In the large N limit [11], the parameter $1/N$ plays the role of an expansion parameter. Physical observables are expected to be smooth functions of λ and $\phi/(Nf)$, which are kept finite in taking the limit.

From this large N scaling argument, we expect that the potential of ϕ, i.e. the θ-angle dependence of the vacuum energy, takes the form

$$V(\phi) = N^2\Lambda^4 \mathsf{V}\left(\frac{\phi}{Nf}\right) + O(N^0), \tag{7}$$

where $\mathsf{V}(x)$ is a smooth function of $O(N^0)$ when written in terms of x. This potential, however, does not respect the expected symmetry under $\phi \to \phi + 2\pi f$. In the large N limit, this transformation induces an infinitesimal shift in the argument of function V, which can be an invariance of the potential $V(\phi)$ only if V is constant. However, this is inconsistent with perturbative large N calculation, which shows otherwise.

The way around this problem is to realize that the potential is multi-valued [10]. In particular, we have many different (in general metastable) branches corresponding to the shift $\phi \to 2\pi f n$ with n integer. The correct vacuum energy, for example, is then given by the minimal values among these branches

$$V_{\rm min}(\phi) = N^2\Lambda^4 \mathrm{min}_n \mathsf{V}\left(\frac{\phi + 2\pi f n}{Nf}\right), \tag{8}$$

so that the invariance of physics under $\phi \to \phi + 2\pi f$ is recovered.

Let us now come to finite values of N. In the large N analysis the value of N is taken to be infinity, so that we have an infinitely many branches, i.e. n runs over all integers in Eq. (8).[2] However, the situation can be different for a finite value of N—if n is taken to be of order N then the shift $\phi \to \phi + 2\pi f n$ changes the argument of $\mathsf{V}(\phi/(Nf))$ by an $O(1)$ amount, which can preserve the value of the function $\mathsf{V}(\phi/(Nf))$. If this happens, there will be only a finite number of metastable branches, with each branch being periodic with the period of $O(2\pi Nf)$.

That a finite number (order $O(N)$) of branches exists is discussed in the analysis of the chiral Lagrangian for QCD (with flavors) in Ref. [10]. The analysis there is justified for small quark masses, whereas here we are interested in the opposite limit of pure Yang–Mills theory, in which the quark masses are taken to be infinitely large.

In the case of pure Yang–Mills theory, we expect that the number of metastable branches is N (so that the periodicity of the θ-dependent potential in a single branch is $2\pi N$, not 2π). This is suggested for example by the analysis of softly-broken $\mathcal{N} = 1$ supersymmetric Yang–Mills theories (see Refs. [13,14]). More recently, this $2\pi N$ periodicity of the θ angle has been made manifest

[2] See, e.g., Refs. [3,12–16] for related discussion in the context of inflation.