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Evaluating longitudinal vibration characteristics of bonded strands embedded in prestressed concrete beams by a system identification approach

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ABSTRACT

This study is to investigate the wire rope governing equation in order to establish the longitudinal vibration characteristics of bonded PSC strands. For this purpose, the longitudinal vibration tests have been conducted for six bonded PSC specimens with the different stress levels. Subsequently, the wave velocities of the strands have been estimated from the test results. Then, a sensitivity-based system identification algorithm has been applied to evaluate the constitutive constants of the strands using the measured wave velocities. The present study demonstrates that the longitudinal dynamic behaviour of the strands is governed by coupled extensional-torsional oscillations. The study also reveals that longitudinal elastic wave velocity of not only the bonded strands but also unbounded strands is nonlinearly increased as the applied tensile stress level increases.

1. Introduction

Recent development of high-strength steel strands has increased the use of prestressed concrete (referred to as PSC hereafter) in the construction of engineering structures. The PSC beam system is a system that introduces compression on concrete in advance using steel tendons. The system accordingly enhances the behaviour of concrete in tension and is also effective in vertical deflection control. This, however, leads to a relatively high level of stress on tendons embedded in the PSC system under normal condition. Subsequently, a tie-breaking loss of prestress force on tendons may prejudice the overall stability of the system. The loss of prestress force is in general classified into immediate and time dependent one. The former represents elastic loss, anchorage loss and frictional loss, while the latter stands for creep, shrinkage and relaxation. Due to both losses, the level of prestressed force is continually reduced on the PSC system [1]. Therefore, periodic (preferably real-time) monitoring is necessary to estimate the loss level of prestress force on tendons. To date, whereas many of theoretical methodologies have been proposed with regard to the monitoring of prestressed steel tendons, no practical methodology that is capable of evaluating nondestructively the level of prestress force on the tendons has been reported [2].

Aforementioned theoretical methodologies have been studied for both bonded and unbonded PSC system. As for the former, cement grouting is injected in between tendons and duct, resulting in perfect bond between tendons and concrete. Meanwhile, the latter injects the grease instead of cement grouting within the duct and has been popularly used in USA and Europe. Particular notification is placed on the fact that most of previous methodologies regarding the tendon force estimation have been focused on the unbounded PSC system. Saiidi et el. [3] have reported that frequency in flexural mode is changed according to the level of prestress force in an unbonded PSC girder. Law and Lu [4] has proposed a scheme for the estimation of prestress force and flexural stiffness using measured deflections and strains in an unbonded PSC girder. However, these are rather limited for practical use and are not desirable for the estimation of prestress force on an individual tendon since they make use of overall response of the PSC system.

To get over the above, Chen and Wissawapaissal [5,6] have experimentally measured the arrival time shift of guided ultrasonic wave on unbonded PSC strands using a pulse-echo technique. Experimental results show that the applied stress level on unbonded PSC strands is linearly proportional to the travelling time of the stress wave propagating inside the centre wire of seven-wire prestressing strands. They have explained that this phenomenon is due to accoustoelastic effect that the ultrasonic velocity is changed with applied stress. Similar researches have been conducted by Washer et al. [7], Scalea et al. [8], Rizzo [9], and Chaki and Bourse [10] and the above phenomenon has been confirmed from these

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researches. Whereas the theoretical basis for such accoustoelastic effect has been originated from the second-order deformation theory, it has been validated only for a homogeneous isotropic infinite cylinder till now [11]. For the case of a finite strand in particular, severe interactions between center wire and the circumferential wires are expected. However, no theoretical investigation for such complicate condition has been presented.

For bonded strands, however, the aforementioned ultrasonic wave technique cannot be applied to estimate stress level by measuring the ultrasonic velocity. This is due to the fact that the energy dissipation of ultrasonic wave from strands to grout is too severe to measure the pulse responses at the end of strands [10]. Beard et al. [12] have reported that the maximum inspection range of ultrasonic wave is approximately 1.5 m. It is however noteworthy that most practical PSC structures have more than a length of 25 m. Therefore a new technique should be developed to evaluate the prestressing level in relatively longer bonded PSC. Kim et al. [13] have firstly investigated the effect of applied stress on the longitudinal vibration characteristics of bonded PSC strands using a pulse-echo technique. It is worthy of noting that a low frequency wave induced by a mechanical impact has much stronger energy than a high frequency wave such as ultrasonic wave. Experimental results show that the longitudinal frequencies of bonded strands are nonlinearly proportional to the applied stresses. However, the mechanism regarding the longitudinal vibration of the bonded strands has not been clearly identified yet.

In line of the above, the objective of this study is to establish the longitudinal vibration mechanism of the bonded PSC strands. To achieve such a goal, theory of a wire rope has been employed for the investigation of the longitudinal vibration of the bonded PSC strands. In addition, longitudinal vibration tests have been carried out for six bonded PSC specimens with different prestressing levels. Then, a sensitivity-based system identification algorithm is newly developed in order to identify system parameters from the experimental test data. Subsequently, concluding remarks are discussed regarding both applicability and accuracy of the present approaches.

2. Longitudinal vibration mechanism of prestressing strands

Since a typical strand consists of several helically wound wires, it is intuitively clear from the geometry that applied tensile load produces not only an extension but also a rotation. In addition, torsional rigidity of the strand increases as the applied tensile stress increases. When the applied tensile stress level is low, individual wires may slip together. However, no slip condition occurs when the applied tensile stress level is relatively high. Thus, the strand may behave as a rod in such condition.

Shown in Fig. 1 is a structural segment of a strand. The symbols f and q denote an external force and an external moment per unit length, respectively. The measure of stressed length is given by S. The reactions of the strand to the external loadings are its tension T and torque C. Samras et al. [14] reveal that a wire rope follows coupled extensional-torsional oscillations. The governing equation for a longitudinal motion is as follows:

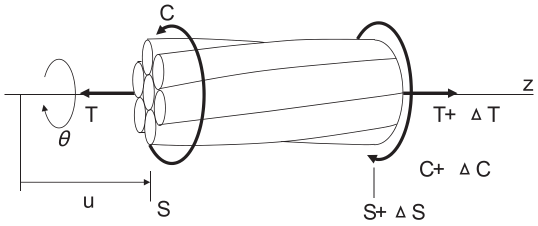

Fig. 1. Structural segment of a strand.

$$\mathrm{A}_1\frac{\partial^2 u}{\partial z^2}+\mathrm{A}_2\frac{\partial^2\theta}{\partial z^2}=m\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\mathrm{A}_3\frac{\partial^2 u}{\partial z^2}+\mathrm{A}_4\frac{\partial^2\theta}{\partial z^2}=I\frac{\partial^2 u}{\partial t^2} \tag{2}$$

where u and θ denote axial deformation and rotation with regard to z-axis, respectively. Terms, m and I represent mass and mass moment of inertia per unit of length, respectively. Constitutive constants, A_1, A_2, A_3, and A_4, stand for stiffness components dependent on both rope material and construction. It is worthy of mentioning that the above governing equations under forced vibration condition have been confirmed by Jiang et al. [15], Raoof et al. [16], and Yen et al. [17] in terms of various closed form solutions.

Both torsional and longitudinal wave speeds denoted as c_1 and c_2, respectively are given by a function of the four constitutive constants, regardless of boundary conditions and thus written as below:

$$c_{1,2}=\frac{2(\mathrm{A}_1\mathrm{A}_4-\mathrm{A}_2\mathrm{A}_3)}{(\mathrm{A}_1 I+\mathrm{A}_4 m)\pm\sqrt{(\mathrm{A}_1 I-\mathrm{A}_4 m)^2+4mI\mathrm{A}_2\mathrm{A}_3}} \tag{3}$$

As for a special case of the clamped ends, torsional and longitudinal frequencies denoted as f_n^1 and f_n^2, respectively are obtained by

$$f_n^1=\frac{n}{2L}c_1 \tag{4}$$

$$f_n^2=\frac{n}{2L}c_2 \tag{5}$$

where n and L represent a mode number and a length of wire rope, respectively. It can be known from Eqs. (4) and (5) that torsional and longitudinal modes occur simultaneously because of a coupling effect. In case that natural frequencies with the clamped boundary conditions are measured, the wave speeds can be obtained as follows:

$$c_1=2L\frac{f_n^1}{n} \tag{6}$$

$$c_2=2L\frac{f_n^2}{n} \tag{7}$$

The constitutive equations for a rope having an introduced tension of T and a torque of C can be given as

$$T=\mathrm{A}_1\varepsilon+\mathrm{A}_2\tau \tag{8}$$

$$C=\mathrm{A}_3\varepsilon+\mathrm{A}_4\tau \tag{9}$$

where ε and τ represent engineering strain and twist per unit length, respectively. Whereas A_1 is dominated by axial stiffness in longitudinal direction, A_4 is mainly governed by torsional stiffness of a wire. Samras et al. [14] pointed out further that the constitutive constants of A_2 and A_3 are identical in terms of Maxwell's reciprocal theorem Eqs. (10) and (11) should be satisfied in order to maintain positive definite strain energy.

$$\mathrm{A}_2=\mathrm{A}_3 \tag{10}$$

$$\mathrm{A}_1\mathrm{A}_4\pm\mathrm{A}_2\mathrm{A}_3>0 \tag{11}$$

With regard to the above four constants, many of theoretical values have been proposed by numerous researchers for full slip condition (i.e. ignoring an inter-wire friction) [18]. However, very few studies have been made for the effect of the inter-wire friction on the four constants. Work done by Raoof and Kraincamic [19] is one of very few studies. They have demonstrated experimentally that effective axial stiffness of a wire-rope is not a constant due to an inter-wire friction and thus changes from no slip to full slip