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Scalar charges and the first law of black hole thermodynamics

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ABSTRACT

We present a variational formulation of Einstein–Maxwell-dilaton theory in flat spacetime, when the asymptotic value of the scalar field is not fixed. We obtain the boundary terms that make the variational principle well posed and then compute the finite gravitational action and corresponding Brown–York stress tensor. We show that the total energy has a new contribution that depends on the asymptotic value of the scalar field and discuss the role of scalar charges for the first law of thermodynamics. We also extend our analysis to hairy black holes in Anti-de Sitter spacetime and investigate the thermodynamics of an exact solution that breaks the conformal symmetry of the boundary.

1. Introduction

Scalar fields play a central role in particle physics and cosmology and arise naturally in the high energy physics unification theories. It is then important to understand generic properties of gravity theories coupled to scalars (and other matter fields), particularly the role of scalars for black hole physics.[1]

The low energy effective actions of string theory correspond to (consistent truncations of) gauged supergravities (see, e.g., [7] and references therein) and some of the accepted wisdoms of general relativity may be reconsidered in this context. One of the important differences is that, contrary to fixing the boundary conditions as in general relativity, the boundary conditions in string theory are determined by dynamical vacuum expectation values (VEVs) of the moduli. An important and unusual consequence is that, for the non-extremal black holes in string theory, both the mass and area of the horizon depend in a non-trivial way on the asymptotic values of the moduli ϕ^a_∞ (here, a labels different scalars), which leads to a drastic modification of the first law of (static) black hole thermodynamics [8]:

$$dM = TdS + \Psi dQ + \Upsilon dP + \left(\frac{\partial M}{\partial \phi^a_\infty}\right) d\phi^a_\infty \tag{1}$$

where Ψ and Υ are the electric and magnetic potentials, and the coefficients of ϕ^a_∞ are computed at fixed charges and entropy,

$$\left(\frac{\partial M}{\partial \phi^a_\infty}\right)_{S,Q,P} = -G_{ab}(\phi_\infty)\Sigma^b \tag{2}$$

With the notations of [8], $G_{ab}(\phi_\infty)$ is the metric of the moduli space and Σ^a are the scalar charges that can be read of from the asymptotic expansion of the scalar field at the spatial infinity:

$$\phi^a = \phi^a_\infty + \frac{\Sigma^a}{r} + O(\frac{1}{r^2}) \tag{3}$$

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[1] Some recent interesting applications can be found in [1–6].

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A similar proposal appears in the context of AdS/CFT duality where, for an exact hairy black hole solution that is asymptotically AdS, it was found that the first law should be modified by an additional conjugate pair (X, Y) of thermodynamic variables [9]:

$$dM = TdS + \Psi dQ + \Upsilon dP + XdY \tag{4}$$

These quantities, (X, Y), are expressible as functions of the conserved charges (M, P, Q) and were interpreted in their own right as a scalar charge and its conjugate potential [9].

One problem with the first law of thermodynamics (1) for stringy black holes is that the scalar charges are not conserved charges. They correspond to degrees of freedom living outside the horizon (the 'hair') and are not associated to a new independent integration constant, that is why it is called 'secondary hair'. In string theory, the scalar fields (moduli) are interpreted as local coupling constants and so a variation of their boundary values is equivalent to changing the couplings of the theory. A resolution was proposed in [10] (see, also, [11]): one can in principle redefine the charges so that the mass and scalar charges do not depend on ϕ_∞, but the price to pay is that the new 'dressed electric and magnetic charges' are not the physical ones. If the asymptotic value of the scalar field is non-zero, but fixed directly in the action, $\phi_\infty = const.$, that corresponds to a different theory with a different coupling (the factor e^{ϕ_∞} is absorbed in the coupling constant not in the values of the charges) for the gauge field and, within that theory, the term $\Sigma d\phi_\infty$ vanishes. This proposal was made concrete in [12] where, by using the 'phase space method', it was shown that this is a valid integrability condition and there is no need for an extra contribution of the scalar field in the first law.

However, we would like to emphasize that the proposal of Gibbons, Kallosh, and Kol [8] is about a variation of the boundary conditions for the scalar fields and so, despite the arguments in [10,12], it stays robust and intriguing. The main question, which still remains, is then why the scalar charges that act as sources for the moduli, but are not conserved, appear in the first law of black hole thermodynamics when considering variations of ϕ_∞?

In this work, we investigate the role of non-trivial boundary conditions of the scalar field in Einstein–Maxwell-dilaton theory. We are interested in asymptotically flat hairy black hole solutions for which the asymptotic value of the scalar can vary and asymptotically AdS dyonic hairy black hole solutions for which the scalar breaks the conformal symmetry of the boundary. In flat spacetime, we obtain a well-posed variational principle by adding a new boundary term to the action, which permits us to compute the correct total energy, and clarify the role of the (non-conserved) scalar charges to the first law of thermodynamics [8]. Armed with the intuition from flat spacetime, we show that once the energy is also correctly obtained in AdS spacetime [13], when the boundary conditions of the scalar field do not preserve the isometries of AdS at the boundary [14], the first law is satisfied and there is no need to consider an extra contribution from the scalar field. These considerations are of special interest when considering the embedding in string theory and the scalar field (dilaton) becomes dynamical and for AdS holographic applications, e.g. the hairy black holes can be used to describe symmetry breaking or phase transitions in the dual quantum field theory.

2. Asymptotically flat hairy black holes

In this section, we propose a variational principle for asymptotically flat hairy black holes when the boundary values of the scalar fields can vary and show that the total energy has a new contribution that is relevant for thermodynamics. The goal of this section is to discuss this issue concretely in the simplest possible non-trivial setting, namely we are going to use the quasilocal formalism of Brown and York [15] for a theory with only one scalar field that is coupled to the gauge field.

2.1. The first law of hairy black hole thermodynamics

We start with a brief review of [8] and, for clarity, we explicitly obtain the scalar charge term in the first law for an exact hairy black hole solution. Besides the graviton, every string theory contains another universal state, a massless scalar called the dilaton ϕ. We consider the Einstein–Maxwell-dilaton action

$$I\left[g_{\mu\nu}, A_\mu, \phi\right] = I_{bulk} + I_{GH} = \frac{1}{2\kappa}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(R - e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu} - 2\partial_\mu\phi\partial^\mu\phi\right) + \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\sqrt{-h}K \tag{5}$$

where $\kappa = 8\pi G_N$ and, with our conventions, $G_N = 1$. The second term is the Gibbons–Hawking boundary term and K is the trace of the extrinsic curvature K_{ab} defined on the boundary $\partial\mathcal{M}$ with the induced metric h_{ab}.

The coupling between the scalar field and gauge field in the action (5) appears in the low energy actions of string theory for particular values of α, though in our analysis we are going to keep α arbitrary. The equations of motion for the metric, scalar, and gauge field are

$$E_{\mu\nu} := R_{\mu\nu} - 2\partial_\mu\phi\partial_\nu\phi - 2e^{\alpha\phi}\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) = 0 \tag{6}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) - \frac{1}{4}\alpha e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu} = 0 \tag{7}$$

$$\partial_\mu\left(\sqrt{-g}e^{\alpha\phi}F^{\mu\nu}\right) = 0 \tag{8}$$

The general metric ansatz for a static dyonic hairy black hole solution is

$$ds^2 = -a^2dt^2 + a^{-2}dr^2 + b^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{9}$$

where the metric functions have only radial dependence, $a = a(r)$ and $b = b(r)$. The gauge field compatible with this ansatz and the equations of motion is