Physics Letters B 782 (2018) 61–68

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Quantum measurement in two-dimensional conformal field theories: Application to quantum energy teleportation

Wu-Zhong Guo [a], Feng-Li Lin [b,c,*,1]

[a] Physics Division, National Center for Theoretical Sciences, National Tsing-Hua University, Hsinchu 30013, Taiwan
[b] Department of Physics, National Taiwan Normal University, Taipei, 116, Taiwan
[c] Department of Physics, California Institute of Technology, Pasadena, CA 91125, USA

ARTICLE INFO

Article history:
Received 22 April 2018
Accepted 4 May 2018
Available online xxxx
Editor: N. Lambert

ABSTRACT

We construct a set of quasi-local measurement operators in 2D CFT, and then use them to proceed the quantum energy teleportation (QET) protocol and show it is viable. These measurement operators are constructed out of the projectors constructed from shadow operators, but further acting on the product of two spatially separated primary fields. They are equivalently the OPE blocks in the large central charge limit up to some UV-cutoff dependent normalization but the associated probabilities of outcomes are UV-cutoff independent. We then adopt these quantum measurement operators to show that the QET protocol is viable in general. We also check the CHSH inequality a la OPE blocks.

1. Introduction

Quantum entanglement has been studied intensively in the past few years in quantum field theory (QFT) and many-body systems, partly inspired by the Ryu–Takayanagi formula of the holographic entanglement entropy [1,2], partly inspired by the new quantum order in the many-body condensed matter systems [3,4], and moreover by the connection of these twos [5,6]. There are usually two ways to characterize the quantum entanglement. One is to evaluate the entanglement entropy or Rényi entropies of the reduced density matrix of a quantum state. The other way is to treat the entanglement of quantum state as the resources for some quantum information tasks, which will help to enhance the efficiency of the similar tasks in the classical computation and communication, and to reduce the complexity. There are many classic examples in the earlier development of quantum information sciences, such as quantum teleportation [7], dense coding [8] and so on. However, most of these examples are performed for the few-qubit systems, and seldom for the QFT or many-body systems.

In this work, we would like to explore the possibility of defining the quantum measurement operators in one of the special QFTs, i.e., the conformal field theory (CFT), so that one can generalize the quantum protocols for qubit systems to the ones in CFT. Some earlier effort along this direction can be found in [9] for free QFT. Here our CFT is in general interacting theory and can be seen as the critical phases of many-body systems. Thus, our scheme can be thought as a precursor to perform the quantum information tasks in critical systems. For simplicity, we will apply our quantum measurement operators in CFT to one particular quantum information task, the so-called quantum energy teleportation (QET) [10–13], for which Alice will send the energy (not the quantum state) to Bob by LOCC. Note that a holographic version for holographic CFT has been studied in [14] based on the so-called surface/state correspondence [15,16], which states that each (space-like) hypersurface in AdS space corresponds to a quantum state in the dual CFT.

We propose that the OPE blocks formulated in [17] can be used as a set of local quantum measurements in the weak sense, i.e., just holds for ground state but not in the operator sense. The OPE blocks can be shown to be equivalent to be the projector operators P_k's with k labeling the outcomes, which are constructed in the shadow formalism [18], acting on the product of two separated local primary operators, i.e., $O_i(x_1)O_j(x_2)$. The projectors P_k's are not local but smear over the entire spacetime. However, in 2D CFTs they can be reduced to quasi-local ones over the causal diamond subtended by the interval $[x_1, x_2]$. The reason of the weak sense is that the set of OPE blocks cannot be complete. This is easy to see by the fact that the set of projectors constructed by shadow formalism is by construction complete, but the associated OPE blocks cannot be. Despite that, this is good enough to adopt them for the QET protocol by either in the weak sense or adopting the view of

* Corresponding author.
E-mail addresses: wzguo@cts.nthu.edu.tw (W.-Z. Guo), fengli.lin@gmail.com (F.-L. Lin).
[1] On leave from NTNU.

https://doi.org/10.1016/j.physletb.2018.05.007

acting P_k's on the excited state $O_i(x_1)O_j(x_2)|0\rangle$ initially prepared by Alice for QET, where $|0\rangle$ is CFT's ground state.

As an application of these OPE block quantum measurement, we adopt them to proceed the QET protocols in 2D CFTs. We find that such QET task is viable. This encourages the experimental realization of QET in 2D critical systems such as the edge states of Quantum Hall Effect. Moreover, we also use these measurement operators to show that one cannot violate CHSH inequality.

In the following the paper is organized as follows. In section 2 we will review the issues of POVM in QFT, and then propose the OPE blocks as the set of quasi-local quantum measurements in 2D CFTs. In section 3 we adopt the OPE blocks as the quantum measurements for the QET protocol and calculate the energetics at each step. We first show that the QET will fail in the infinite time limit, and then show that the sub-leading correction beyond this limit will then yield QET by appropriate quantum feedback control. Finally, we give a toy example for demonstration of viable QET in CFT. We then conclude our paper in section 4 and end with a discussion on Bell inequality of the OPE blocks.

2. Projection measurements in CFT

A quantum measurement process can be described by a set of positive operators $\{E_k\}$ whose sum is the identity operator, i.e.,

$$\sum_k E_k = \mathbb{I}\,. \tag{1}$$

Then, the probability of obtaining the outcome k when measuring the state $|\psi\rangle$ is

$$\mathrm{p}_k = \langle\psi|E_k|\psi\rangle\,. \tag{2}$$

This is known as the positive operator-valued measure (POVM). A special case is when the positive operators E_k's are all projection operators, i.e., $E_k^\dagger E_j = \delta_{k,j}E_k$, then the normalized post-measurement state of outcome k is

$$|\psi_k\rangle = \frac{E_k|\psi\rangle}{\sqrt{\langle\psi|E_k|\psi\rangle}}\,. \tag{3}$$

This is the so-called projective-valued measure (PVM).

Moreover, the POVM can also be constructed by introducing the auxiliary probe coupled to the state $|\psi\rangle$, so that the operator E_k can be obtained as follows: acting on the total system by the time evolution operator $U(t)$, and then projecting it onto the probe's eigenstate $|k\rangle_p$, i.e.,

$$E_k := M_k^\dagger M_k \tag{4}$$

with

$$M_k := {}_p\langle k|U(t)|0\rangle_p\,, \tag{5}$$

where the subscript p denotes "probe". It is easy to see that (1) is satisfied by $U^\dagger U = 1$.

Based on the above procedure, one may construct the POVM in quantum field theory (QFT) and then implement them on some quantum tasks, see for example [13] on constructing POVM of free QFT for quantum energy teleportation. However, in practical the construction of POVM for interacting QFT is not so straightforward due to nontrivial operator mixings.

2.1. OPE block in CFT

Instead, in d-dimensional CFTs there is a set of projection operators constructed by the shadow operator formalism [18], and explicitly they are given by

$$P_k = \frac{\Gamma(\Delta_k)\Gamma(d-\Delta_k)}{\pi^d\Gamma(\Delta_k-\frac{d}{2})\Gamma(\frac{d}{2}-\Delta_k)}\int D^dX\,\mathcal{O}_k(X)|0\rangle\langle 0|\tilde{\mathcal{O}}_k(X), \tag{6}$$

where $\Gamma(x)$ is the Gammas function and Δ_k is the conformal dimension of $\mathcal{O}_k$. These projectors are complete if k runs over all primaries, i.e.,

$$\sum_{k\in\text{all primaries}} P_k = \mathbb{I}_{CFT}\,. \tag{7}$$

We have introduced the shadow operator[2,3]

$$\tilde{\mathcal{O}}_k(X) := \int D^dY\,\frac{1}{(-2X\cdot Y)^{d-\Delta_k}}\mathcal{O}_k(Y), \tag{8}$$

so that it can be used to show that

$$P_iP_j = \delta_{i,j}P_i\,. \tag{9}$$

In the above, we adopt the notation of embedding space for the coordinate X, i.e., for CFT in d-dimensions, the "embedding space" is $\mathbb{R}^{d,2}$. The dimensional space is obtained by quotienting the null cone $X^2=0$ and by the rescaling $X\sim\lambda X$, $\lambda\in\mathbb{R}$. In particular, we can choose the Poincare section such that $X := (X^+, X^-, X^\mu) = (1, x^\mu x_\mu, x^\mu)$ such that

$$-2X_1\cdot X_2 = (x_1-x_2)^2\,.$$

Even though P_j's are projection operators, however, it is not local and thus we cannot use them to implement local quantum measurements which are required in many quantum information tasks such as quantum (energy) teleportation. Fortunately, for 2D CFTs the P_j becomes a quasi-local operator when acting on the following states

$$O_1(x_1)O_2(x_2)|0\rangle, \tag{10}$$

where $|0\rangle$ is the ground state of CFT. In this case, the integration in (6) and (8) is over the casual diamond $\mathcal{D}_{\mathbf{A}}$ subtended by the interval $[x_1, x_2]$, i.e., $x_1 < x_2$ w.l.o.g. For simplicity, we will only consider the case with $O_1 = O_2 := O_i$ the primary operator of conformal dimension $(h_i, \bar{h}_i)$. We can then view the state (10) as some quasi-local excitation prepared by Alice, and then she further performs a local projection measurement within her causal domain for some quantum information task.

Indeed, the post-measurement state is related to the OPE block defined in [17], i.e.,

$$P_k\,\mathcal{O}_i(x_1)\mathcal{O}_i(x_2)|0\rangle = x_{12}^{-2h_i-2\bar{h}_i}c_{iik}\,\mathcal{B}_k(x_1,x_2)|0\rangle\,, \tag{11}$$

where c_{iik} is the OPE coefficient and $x_{mn} := x_m - x_n$. By this definition, it is straightforward to relate the conformal block $g_k(u, v)$ and the two-point correlator of the OPE blocks, i.e.,

[2] In (6), we adopt the notation of embedding space for the coordinate X, i.e., for CFT in d-dimensions, the "embedding space" is $\mathbb{R}^{d,2}$. The dimensional space is obtained by quotienting the null cone $X^2=0$ by the rescaling $X\sim\lambda X$, $\lambda\in\mathbb{R}$. In particular, we can choose the Poincare section such that $X := (X^+, X^-, X^\mu) = (1, x^\mu x_\mu, x^\mu)$ such that

$$-2X_1\cdot X_2 = (x_1-x_2)^2\,.$$

[3] The "conformal integral" in (8) is defined by [18]

$$\int D^dX\,f(X) = \frac{1}{\text{Vol GL}(1,\mathbb{R})^+}\int_{X^++X^-\geq 0} d^{d+2}X\,\delta(X^2)\,f(X)\,.$$