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Knot polynomials for twist satellites

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ABSTRACT

We begin the systematic study of knot polynomials for the twist satellites of a knot, when its strand is substituted by a 2-strand twist knot. This is a generalization of cabling (torus satellites), when the substitute of the strand was a torus knot. We describe a general decomposition of satellite's colored HOMFLY in those of the original knot, where contributing are adjoint and other representations from the "E_8-sector", what makes the story closely related to Vogel's universality. We also point out a problem with lifting the decomposition rule to the level of superpolynomials — it looks like such rule, if any, should be different for positive and negative twistings.

1. Introduction

Knot polynomials [1] are the only observables in the Chern–Simons theory [2]. They are at once the simplest among the Wilson loop averages in gauge theories and among explicit realizations of dualities (modular transformations) in conformal theories. They are essentially non-perturbative and reveal the relevance of topological-field-theory description of non-perturbative physics. They can be explicitly calculated — and thus provide a set of exactly solvable examples for all above-mentioned topics, which are undoubtedly the central ones of today's theoretical physics.

Still, the theory of knot polynomials is making just its first steps, and explicit calculations are still available only for very restricted classes of knots. Any extension of these classes is valuable and can help to identify the general properties of physical observables and build their alternative descriptions, which do not refer to gauge theory formalism and can be used in confinement phases. In this letter we suggest one of such awaited extensions: from any previously-studied knot to its k-twisted satellite, which is usually a knot of a far more complicated structure. Still, as we demonstrate, its knot polynomials can be directly reduced to those of the original knot, i.e. we can actually build an infinite sequence

$$\text{knot} \longrightarrow \text{its satellites} \longrightarrow \text{satellites of these satellites} \longrightarrow \ldots$$

leading to knots of arbitrary high complexity. This letter is just the first step, but the idea should be clear — as well as the many challenges and problems it can raise and resolve. For new steps along these lines see [3].

2. Whitehead doubles

The k-twist satellite, also known as Whitehead double, $S_k(\mathcal{K})$ of $\mathcal{K}$ is defined [4] as the image of the twist knot Twist_k, naturally embedded into a $3d$ solid torus, which is then identified with the tubular neighborhood of the knot $\mathcal{K}$:

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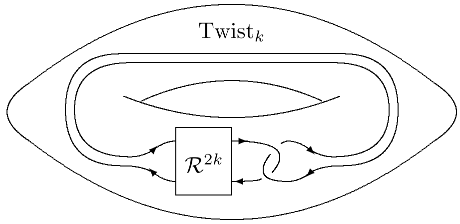
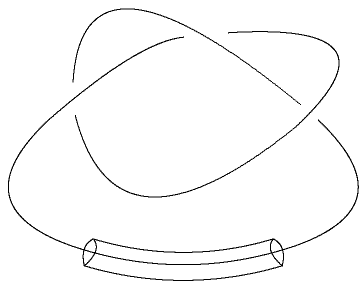
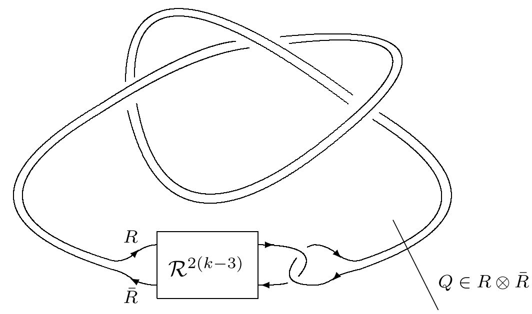




$\mathcal{K} = 3_1 =$ trefoil
with a fragment of its tubular neighborhood



$S_k(3_1) = k-$twist satellite of the trefoil
with $w^{3_1'} = -3$

Twist_k and $\mathcal{K}$ are often called respectively the *pattern* and the *companion* of the satellite $S_k(\mathcal{K})$. Likewise one can define satellites of other types, e.g. with patterns which are torus knots (satellites in this case would be just the ordinary *cablings* [5] of $\mathcal{K}$), but in this letter we concentrate on the k-twist ones.

3. HOMFLY polynomials for satellites

According to [6], reduced HOMFLY polynomial in representation Q for a knot $\mathcal{K}$, described as a closure of an m-strand braid is given by

$$\mathcal{H}_Q^{\mathcal{K}} = \sum_{Y \in Q^{\otimes m}} D_Y \cdot \mathrm{Tr}_{\mathrm{mult}_Y} \mathcal{R}_Y^{\mathcal{K}} \tag{1}$$

where $\mathcal{R}_Y$ is a convolution of $\mathcal{R}$-matrices in Tanaka–Krein representation, which are square matrices of the size $\mathrm{mult}_Y \times \mathrm{mult}_Y$ and mult_Y is the multiplicity of representation Y in the decomposition of the product

$$Q^{\otimes m} = \oplus_Y \mathrm{mult}_Y \cdot Y \tag{2}$$

Now, the satellite $S_k(\mathcal{K})$ is obtained by substituting the knot by a 2-wire antiparallel cable, carrying representation $R \otimes \bar{R}$, and by changing one of the m traces for a cut of the twist knot $\mathrm{Twist}_{k+w^{\mathcal{K}'}}$. Here $w^{\mathcal{K}'}$ is the writhe number of $\mathcal{K}'$, which actually depends on the $2d$ knot diagram, not just on the knot — we put prime over $\mathcal{K}$ to remind about this diagram dependence. In what follows we often write just w instead of $w^{\mathcal{K}'}$ to simplify the formulas. Strictly speaking the satellite itself also depends on the knot diagram, but this dependence can be compensated by the shift of k, see (25) below for a simple example — and we omit prime in $S_k(\mathcal{K}')$.