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An analytic effective model for hairy black holes

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ABSTRACT

Hairy black holes (BHs) have macroscopic degrees of freedom which are not associated with a Gauss law. As such, these degrees of freedom are not manifest as quasi-local quantities computed at the horizon. This suggests conceiving hairy BHs as an interacting system with two components: a "bald" horizon coupled to a "hairy" environment. Based on this idea we suggest an effective model for hairy BHs – typically described by numerical solutions – that allows computing *analytically* thermodynamic and other quantities of the hairy BH in terms of a fiducial bald BH. The effective model is *universal* in the sense that it is only sensitive to the fiducial BH, but not to the details of the hairy BH. Consequently, it is only valid in the vicinity of the fiducial BH limit. We discuss, quantitatively, the accuracy of the effective model for asymptotically flat BHs with synchronised hair, both in $D=4$ (including self-interactions) and $D=5$ spacetime dimensions. We also discuss the applicability of the model to synchronised BHs in $D=5$ asymptotically *AdS* and static $D=4$ coloured BHs, exhibiting its limitations.

1. Introduction

The 1970s represent a golden era for the theoretical study of black holes (BHs). At the classical level it was understood that elctrovacuum BHs are remarkably featureless, being completely classified by a small number of *macroscopic* independent degrees of freedom: mass, angular momentum and electric (possibly also magnetic) charge – see [1] for a review. At the quantum level, on the other hand, the visionary works of Bekenstein [2] and Hawking [3] heralded BHs as gateways into the realm of quantum gravity, by showing they are thermodynamical objects, and, in particular that they have an entropy, geometrically computed as the horizon area. Understanding and counting the *microscopic* degrees of freedom associated to this entropy became a primary challenge for any quantum gravity candidate theory. Two decades later, some remarkable success was obtained within String Theory, werein, starting with [4,5], it was possible to identify and count the microscopic degrees of freedom that explain the classical geometric entropy, defined by the BHs macroscopic degrees of freedom, albeit only in some particular classes of BHs.

The macroscopic simplicity of electrovacuum BHs suggested the "no-hair" conjecture [6]: that the endpoint of gravitational collapse is a stationary BH completely described by a small set of macroscopic degrees of freedom, all of which should be associated to Gauss laws. In particular, this means such degrees of freedom are manifest at the horizon and can be computed therein as quasi-local quantities (*e.g.* the Komar integrals [7] associated to the mass and angular momentum of stationary axisymmetric BHs and the Gauss law associated to the electric charge). This, in turn, ties up nicely with the microscopic picture, and the view that the horizon contains all relevant BH information.

The discovery of "hairy" BHs in a variety of models (see *e.g.* [8–10] for reviews) has overshadowed this conceptually simple picture. These BHs have extra macroscopic degrees of freedom not associated to a Gauss law. Therefore they do not seem to be associated to any quasi-local conserved quantity computable at the horizon level. This raises interesting questions, on how the microscopic description of the BH captures these extra macroscopic degrees of freedom, but it also suggests an effective model for obtaining an (in general) analytic approximation for physical and thermodynamical quantities of the hairy BHs associated to the horizon [11].

The basic idea of the effective model sketched in [11] (and suggested by the numerical evolutions in [12]), therein called *quasi-Kerr horizon model*, is that due to the absence of further local charges at the horizon, the horizon of the hairy BH is well approximated by the horizon of a fiducial bald BH but with different parameters. In a sense, the hairy BH can be conceived as a cou-

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pled system of a bald horizon with an external "hair" environment. Naturally, the system is interacting and the non-linearities of the underlying gravity–matter system introduce a non-trivial deformation of the "bald" horizon. But in the feeble hair regime, when only a small percentage of the overall spacetime energy is contained in the matter field, these non-linearities are expected to be small, and the horizon should still behave as that of the bald fiducial BH, but with shifted parameters to take into account the mass and angular momenta that is no longer inside the horizon but rather in the matter environment. One could expect such simple model to yield errors in the thermodynamics quantities of the order of the deviation from the fiducial bald BH. The findings in [11], however, revealed that this effective model gives an unexpectedly good approximation, sometimes with deviations of $\sim \mathcal{O}(1\%)$ even for fairly large deviations from the bald BH, *e.g.*, when $\sim \mathcal{O}(30\%)$ of the spacetime energy is stored in the matter field.

The purpose of this paper is to investigate the applicability and accuracy of the model by considering further examples of hairy BHs. Thus, after reviewing the assumptions, basic statement and corollaries of the effective model in Section 2, we consider two applications in Section 3: we apply it to Kerr BHs with synchronised hair and self-interactions [13] in Section 3.1 and to five dimensional ($D = 5$) Myers–Perry BHs with synchronised hair [14] in Section 3.2. In both these examples the effective model performs well. In the $D = 4$ case the accuracies are comparable to those described at the end of the last paragraph. In the $D = 5$ case, there is a mass gap between the hairy BHs and the fiducial BH. This means that the fiducial BH geometry is never approached globally, but only locally. In this case we find that, even for very hairy BHs, for which $\sim \mathcal{O}(90\%)$ of the spacetime energy is stored in the matter field, the model can yield errors of $\sim \mathcal{O}(1\%)$ for some physical quantities. To exhibit also the limitations of the effective model, we consider in Section 4 two further applications: to the $D = 5$ *AdS* Myers–Perry BHs with synchronised hair [15] and to the coloured BHs in Einstein–Yang–Mills theory [16]. With these applications, we illustrate either difficulties in the formalism, or unimpressive accuracies. In Section 5 we present some final remarks, in particular speculating about the underlying reason for the good accuracy of the model in the case of asymptotically flat BHs with synchronised hair.

2. The general framework

2.1. Komar integrals and Smarr relation

We consider a general model in $D \geqslant 4$ spacetime dimensions, consisting of Einstein's gravity minimally coupled to some matter fields ψ described by a Lagrangian density $\mathcal{L}_m$

$$\mathcal{S} = \int d^D x \sqrt{-g}\left[\frac{R}{16\pi} + \mathcal{L}_m\right] , \tag{1}$$

where R is the spacetime Ricci scalar. Here and below we use geometrised units, setting Newton's constant and the speed of light to unity: $G = 1 = c$.

In this work we shall be interested in stationary space-times with N-azimuthal symmetries, where $N = 1, 2$, for $D = 4, 5$. This implies the existence of $N + 1$ commuting Killing vectors, $\xi \equiv \partial_t$, and $\eta^{(k)} \equiv \partial_{\varphi_k}$, for $k = 1, \ldots, N$.

Assuming asymptotic flatness, the total (or ADM) mass M and total angular momenta $J_{(k)}$ of the configurations are obtained from Komar integrals [7] (see also, *e.g.* [17]), at spatial infinity, associated with the corresponding Killing vector fields

$$M = -\frac{1}{16\pi}\frac{D-2}{D-3}\int_{S^{D-2}_\infty} \alpha \, , \qquad J^{(k)} = \frac{1}{16\pi}\int_{S^{D-2}_\infty} \beta^{(k)} \, , \tag{2}$$

with

$$\begin{aligned} &\alpha_{\mu_1\ldots\mu_{D-2}} \equiv \epsilon_{\mu_1\ldots\mu_{D-2}\rho\sigma}\nabla^\rho \xi^\sigma \, , \\ &\beta^{(k)\mu_1\ldots\mu_{D-2}} \equiv \epsilon_{\mu_1\ldots\mu_{D-2}\rho\sigma}\nabla^\rho \eta^{(k)\sigma} \, . \end{aligned} \tag{3}$$

We are mainly interested in BH solutions with a regular event horizon geometry (without any restrictions on its topology, which for $D > 4$ can be non-spherical [18–20]). This horizon $\mathcal{H}$ has an associated (hyper)area of its spatial sections, A_H, and a temperature T_H; there are also N horizon angular velocities $\Omega_{H(k)}$ associated with the N-azimuthal symmetries.

Using Komar integrals computed at the event horizon, one also defines a horizon mass M_H and a set of N horizon angular momenta $J_H^{(k)}$,

$$M_H = -\frac{1}{16\pi}\frac{D-2}{D-3}\int_{\mathcal{H}} \alpha \, , \quad J_H^{(k)} = \frac{1}{16\pi}\int_{\mathcal{H}} \beta^{(k)} \, . \tag{4}$$

Then the following Smarr type mass formulae [21] hold: for the horizon quantities we have

$$\frac{D-3}{D-2}M_H = \frac{1}{4}T_H A_H + \sum_{(k)} \Omega_{H(k)} J_H^{(k)} , \tag{5}$$

whereas for the bulk quantities

$$M = \frac{D-2}{D-3}\left[\frac{1}{4}T_H A_H + \sum_{(k)} \Omega_{H(k)}(J^{(k)} - J^{(k)}_{(\psi)})\right] + M_{(\psi)} \, . \tag{6}$$

In the above relations, $M_{(\psi)}$, $J^{(k)}_{(\psi)}$ are the energy and angular momenta stored in the matter fields, with

$$M = M_H + M_{(\psi)}, \quad J^{(k)} = J_H^{(k)} + J^{(k)}_{(\psi)} \, . \tag{7}$$

Via the Einstein equations, $M_{(\psi)}$ and $J^{(k)}_{(\psi)}$ can be expressed as volume integrals for the appropriate components of the energy–momentum tensor (see *e.g.* [17]).

In addition to the above Smarr relations, the configuration should satisfy the first law of BH thermodynamics [22],

$$dM = \frac{1}{4G}T_H dA_H + \sum_{(k)} \Omega_{H(k)} dJ^{(k)} + \mathcal{W}, \tag{8}$$

where $\mathcal{W}$ denotes the work term(s) associated with the matter fields. In particular, for vacuum solutions, the following relation holds

$$dM_H = \frac{1}{4}T_H dA_H + \sum_{(k)} \Omega_{H(k)} dJ_H^{(k)} \, . \tag{9}$$

2.2. The effective model

We now turn into the assumptions of the effective model [11], its statement and its corollaries.

Assumption 1 *(Fiducial "bald" BH).* One defines a vacuum *fiducial* BH solution[1] which is approached smoothly as $M_{(\psi)} \to 0$, $J^{(k)}_{(\psi)} \to 0$ (*i.e.* with the same symmetries and horizon structure as the non-vacuum solution). Moreover, at least in all cases discussed in this work, the horizon quantities of the fiducial BH have known (in

[1] In $D = 4$ the fiducial solution is obviously Kerr. But in higher dimensions, there can be different solutions for the same global charges and the horizon topology [18–20], thus requiring the definition of the fiducial solution.