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Large leptonic Dirac CP phase from broken democracy with random perturbations

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ABSTRACT

A large value of the leptonic Dirac CP phase can arise from broken democracy, where the mass matrices are democratic up to small random perturbations. Such perturbations are a natural consequence of broken residual $\mathbb{S}_3$ symmetries that dictate the democratic mass matrices at leading order. With random perturbations, the leptonic Dirac CP phase has a higher probability to attain a value around $\pm\pi/2$. Comparing with the anarchy model, broken democracy can benefit from residual $\mathbb{S}_3$ symmetries, and it can produce much better, realistic predictions for the mass hierarchy, mixing angles, and Dirac CP phase in both quark and lepton sectors. Our approach provides a general framework for a class of models in which a residual symmetry determines the general features at leading order, and where, in the absence of other fundamental principles, the symmetry breaking appears in the form of random perturbations.

1. Introduction

Flavor mixing has been observed in both quark and lepton sectors. Theoretically, there are two basic approaches in explaining the mixing patterns. One is top–down by assigning some full flavor symmetry to constrain the fundamental Lagrangian. However, the full flavor symmetries have to be broken, otherwise the up- and down-type fermions would be subject to the same flavor structure in their mass matrices and hence we obtain the same mixing matrices, $V_u = V_d$, leading to a trivial physical mixing matrix, $V_{\rm CKM} = V_u^\dagger V_d = I$. If the mixing pattern is really determined by symmetry, it has to be a residual symmetry that survives symmetry breaking. The reverse of the top–down logic is the bottom–up *phenomenological mass matrix approach* [1–5]. By reconstructing the residual symmetries in both up- and down-type fermion sectors [6,7], the full flavor symmetry can be obtained as a product group [8–10].

In both approaches, the residual symmetry takes the role of predicting the mixing pattern [11–14]. In some sense, the residual symmetry takes the same role as the custodial symmetry in the gauge sector [15]. The electroweak $SU(2)_L \times U(1)$ gauge theory can predict the existence of four gauge bosons but not the weak mixing angle θ_w. In the Standard Model, the weak mixing angle $\sin\theta_w = g'/\sqrt{g^2 + g'^2}$ is a function of the gauge couplings g and g' whose values cannot be predicted by gauge symmetry. The custodial symmetry can make correlation between physical observables, $\cos\theta_w = M_W/M_Z$. Likewise, residual symmetries can predict the correlation among mixing angles and the Dirac CP phase, also known as *sum rule* of mixing angles [16–18] in addition to the mass sum rules [19,20]. The custodial symmetry is essentially a residual symmetry. In this sense, the concept of residual symmetry and phenomenological mass matrix approach applies universally for all the observed mixing among fundamental particles.

The mixing patterns in the quark and lepton sectors are quite different. While the mixings in the quark sector are small, the lepton sector has large mixing angles. This seems hard to understand at the first glance. Considering the fact that the observed quark and lepton mixings are combined effects of mixings in both up- and down-type fermions, $V_{\rm CKM} = V_u^\dagger V_d$ for quarks and $V_{\rm PMNS} = V_\ell^\dagger V_\nu$ for leptons, a unified picture might be possible if a similar mixing pattern appears in both up- and down-type quark mass matrices but in only one of leptons and neutrinos, $V_u \approx V_d \approx V_\ell$ or V_ν. Then the quark mixing $V_{\rm CKM}$ is close to unity matrix while the neutrino mixing $V_{\rm PMNS} \approx V_\ell$ or V_ν has large mixing angles [4,5,15,21,22].

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Approximate democratic matrix is known as an interesting possibility to explain the large hierarchy in quark masses and the small CKM mixing angles when applied to both up- and down-type quarks. If one applies this hypothesis to the lepton sector for both charged leptons and neutrinos, with the help of residual $\mathbb{S}_3$ symmetries, one may get small lepton mixing angles which is strongly excluded experimentally. However, it was pointed out that if we assume almost diagonal mass matrix for neutrinos we obtain large lepton mixing angles [4,5]. A natural consequence is that $V_{\rm CKM}$ has only 1–2 mixing while $V_{\rm PMNS}$ can have at least two large mixing angles at leading order, as we would elaborate in Sec. 2. The democratic matrix can also explain the mass hierarchies among charged fermions, with $m_1 = m_2 = 0$. To accommodate nonzero fermion masses and to get a better fit for observed mixing angles, one needs deviations from the democratic matrix which break the residual $\mathbb{S}_3$ symmetries. With residual $\mathbb{S}_3$ symmetries broken, there is no fundamental principle to regulate the deviations. A natural approach is to assume that small random perturbations make the mass matrix different from the democratic form. This approach is different from the anarchy model, where the mass matrix can be totally free of any constraint [23,24]. We will elaborate our predictions for both neutrino (see Sec. 3 and Sec. 4) and quark mixings (see Sec. 5).

2. Democratic mass matrix hypothesis – preliminary

The democratic pattern of mass matrix can be realized by applying two independent residual $\mathbb{S}_3^L$ and $\mathbb{S}_3^R$ symmetries to the left- and right-handed fermions. Then the fermion mass matrix in a natural basis looks like

$$M_f = \frac{M_0}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \qquad (2.1)$$

where M_0 characterizes the mass scale [4,5,25–51]. Its diagonalization involves two mixing matrices for the left- and right-handed fermions, $M_f = V_L D_f V_R^\dagger$ where $D_f = \mathrm{diag}\{m_1, m_2, m_3\}$ is the diagonalized mass matrix and V_L (V_R) is the mixing matrix of left-handed (right-handed) fermions.

The democratic mass matrix form (2.1) applies for all fermions, except the neutrinos. This can be naturally realized with $SO(3)_L \times SO(3)_R$ flavor symmetries [5]. The three generations of left- and right-handed fermions form triplets under $SO(3)_L$ and $SO(3)_R$ transformations, respectively. Similarly there are two triplet flavons ϕ_L and ϕ_R, correspondingly. Then, two invariants can be written down to form a Yukawa term

$$\sum_{ij} y_{ij} \left(\overline{\psi}_{L,i} \phi_{L,i} \right) \left(\phi_{R,j} \psi_{R,j} \right) . \qquad (2.2)$$

The $SO(3)_L \times SO(3)_R$ flavor symmetry would break down to the residual $\mathbb{S}_3^L \times \mathbb{S}_3^R$ if the triplet Higgs obtains equal vacuum expectation values for the three components, $\langle \phi_{L,R} \rangle \propto (1,1,1)$, leading to the democratic mass matrix (2.1) for charged fermions. For neutrinos, its mass term can be given by Weinberg–Yanagida operator [52,53] which contains two left-handed fermions, $L_{L,i} L_{L,j}$. Note that these two fermions belong to the same $SO(3)_L$ triplet whose product decomposes as $\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{3} + \mathbf{5}$. Of the three decomposed representations, the triplet $\mathbf{3}$ has anti-symmetric Clebsch–Gordan coefficients which is not consistent with the Majorana neutrino mass matrix. The other two, $\mathbf{1}$ and $\mathbf{5}$, can give symmetric Majorana neutrino mass matrix. The singlet contribution is proportional to a unit matrix and for the $\mathbf{5}$ representation we adopt two scalar multiplets with

$$\Sigma_L^{(1)} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} , \qquad \Sigma_L^{(2)} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \qquad (2.3)$$

to give diagonal neutrino mass matrix. Altogether, the $\mathbf{1}$ and $\mathbf{5}$ representations give a diagonal neutrino mass matrix with the three mass eigenvalues being free.

The concrete form of V_L is determined by $M_f M_f^\dagger = V_L D_f^2 V_L^\dagger$. Note that $M_f M_f^\dagger$ also takes the same form as (2.1), but with M_0 replaced by M_0^2. By diagonalizing $M_f M_f^\dagger$, we can obtain the mixing matrix V_L

$$V_L^\dagger = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix} \begin{pmatrix} c_T & s_T e^{i\phi} & 0 \\ -s_T e^{-i\phi} & c_T & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \equiv R T V_0 , \qquad (2.4)$$

which is the most general form of the solution. For convenience, we denote the mixing angle among the states of degenerated mass eigenvalues $m_1 = m_2 = 0$ as $(c_T, s_T) \equiv (\cos\theta_T, \sin\theta_T)$, to which a Dirac-type CP phase ϕ is attached. In addition, there are three free rephasing degrees of freedom α_i with $i = 1, 2, 3$ that are attached to the three mass eigenvalues. It is interesting to observe that *the democratic mass matrix leads to hierarchical mass eigenvalues.*

Since the same democratic mass matrix applies to both up- and down quarks, the CKM matrix naturally has suppressed 1–3 and 2–3 mixings,

$$V_{\rm CKM} = T_u T_d^\dagger = \begin{pmatrix} c_{T,u} & s_{T,u} e^{i\phi_u} & 0 \\ -s_{T,u} e^{-i\phi_u} & c_{T,u} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c_{T,d} & -s_{T,d} e^{i\phi_d} & 0 \\ s_{T,d} e^{-i\phi_d} & c_{T,d} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \qquad (2.5)$$

For cleanness, we have omitted the two rephasing matrices R_u and R_d. The combined 1–2 mixing takes the form as

$$\cos\theta_{12} = \left| c_{T,u} c_{T,d} + s_{T,u} s_{T,d} e^{i(\phi_u - \phi_d)} \right| \quad \text{and}$$
$$\sin\theta_{12} = \left| c_{T,u} s_{T,d} - s_{T,u} c_{T,d} e^{i(\phi_u - \phi_d)} \right| , \qquad (2.6)$$

while $\theta_{13} = \theta_{23} = 0$. This naturally explains why the mixing in the quark sectors are small.

For the neutrino sector, (2.4) is already the form for the PMNS matrix,

$$V_{\rm PMNS} = \begin{pmatrix} -\frac{c_{T,\ell}}{\sqrt{2}} - \frac{s_{T,\ell} e^{i\phi_\ell}}{\sqrt{6}} & \frac{c_{T,\ell}}{\sqrt{2}} - \frac{s_{T,\ell} e^{i\phi_\ell}}{\sqrt{6}} & \frac{2 s_{T,\ell} e^{i\phi_\ell}}{\sqrt{6}} \\ \frac{s_{T,\ell} e^{-i\phi_\ell}}{\sqrt{2}} - \frac{c_{T,\ell}}{\sqrt{6}} & -\frac{s_{T,\ell} e^{-i\phi_\ell}}{\sqrt{2}} - \frac{c_{T,\ell}}{\sqrt{6}} & \frac{2 c_{T,\ell}}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} , \qquad (2.7)$$

assuming the neutrino mass matrix is diagonal, i.e., $V_\ell = I$. Comparing with the standard parametrization of the PMNS matrix we can obtain

$$\sin\theta_r = \frac{2 s_{T,\ell}}{\sqrt{6}} , \qquad \tan\theta_a = \sqrt{2} c_{T,\ell} ,$$
$$\tan\theta_s = \left| \frac{\sqrt{3} c_{T,\ell} - s_{T,\ell} e^{i\phi_\ell}}{\sqrt{3} c_{T,\ell} + s_{T,\ell} e^{i\phi_\ell}} \right| , \qquad \delta_D = -\phi_\ell . \qquad (2.8)$$

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