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Physics Letters B
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Black holes in massive gravity as heat engines

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ARTICLE INFO

Article history:
Received 23 February 2018
Received in revised form 25 March 2018
Accepted 26 March 2018
Available online xxxx
Editor: N. Lambert

ABSTRACT

The paper at hand studies the heat engine provided by black holes in the presence of massive gravity. The main motivation is to investigate the effects of massive gravity on different properties of the heat engine. It will be shown that massive gravity parameters modify the efficiency of engine on a significant level. Furthermore, it will be shown that it is possible to have a heat engine for non-spherical black holes in massive gravity, and therefore, we will study the effects of horizon topology on the properties of heat engine. Surprisingly, it will be shown that the highest efficiency for the heat engine belongs to black holes with the hyperbolic horizon, while the lowest one belongs to the spherical black holes.

1. Introduction

Einstein's General Relativity (henceforth GR) is one of the successful theories for describing gravity since it has been introduced. This theory has some interesting predictions, such as the existence of gravitational waves which recently were detected by the advanced LIGO [1]. Despite its success, there are some phenomena which GR cannot explain precisely. For example, we refer the reader to the current accelerated expansion of the universe. In addition, this theory predicts the existence of massless spin-2 gravitons in which they have two degrees of freedom. However, there have been some arguments regarding the possibility of the existence of massive spin-2 gravitons, such as the hierarchy problem and also brane-world gravity solutions (see Refs. [2] for more details). These studies show that despite its correctness, GR is not a complete theory of gravitation. On the other hand, regarding the massive gravitons lead to some interesting properties in various aspects of gravity. One of them is that this theory and its extensions could explain the accelerated expansion of the universe without considering the dark energy [3,4]. Also, the graviton could behave like a lattice excitation and exhibits a Drude peak in this theory of gravity. It is notable that, current experimental data from the observations of gravitational waves by advanced LIGO require the mass of graviton to be smaller than the inverse period of orbital motion of the binary system, that is $m = 1.2 \times 10^{-22} ev/c^2$ [1].

Historically speaking, Fierz and Pauli were the first ones to introduce a massive theory of gravity in a flat background (FP theory) [5]. The problem with this theory was the fact that it suffers from vDVZ (van Dam–Veltman–Zakharov) discontinuity. Indeed, FP theory had an important problem which is related to the fact that this theory could not recover GR. In other words, FP theory leads to physical predictions so different from those of GR, such as calculating the bending of light around the Sun. The obtained results of light bending around the Sun is only 75% of the obtained result from GR. To resolve this problem, Vainshtein introduced his well-known mechanism requiring the theory being considered in a nonlinear framework. He showed that the solutions of the linearized nonlinear FP theory were only valid at distances larger than a length scale, the Vainshtein radius R_V, which goes to infinity when $m \to 0$. Also, he showed that there exists a well-behaved expansion valid at distances smaller than R_V. More investigations on the Vainshtein mechanism in order to couple and recovery of GR in massive gravity have been done in Refs. [6]. Although Vainshtein mechanism was a solution to vDVZ discontinuity, it was shown that FP theory encounters yet another profound problem known as Boulware–Deser ghost [7] which signals instability in the theory of interest. In order to avoid such instability, sev-

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https://doi.org/10.1016/j.physletb.2018.03.072
0370-2693/

eral models of massive theory are introduced [8,9]. For example, Bergshoeff, Hohm and Townsend proposed one of the interesting models of massive theories in three dimensional spacetime which is known as new massive gravity [8]. This theory of massive gravity and its interesting properties have been investigated in literature [10–13]. However, this theory has some problems in higher dimensions. Another interesting class of massive gravity was introduced by de Rham, Gabadadze and Tolley (dRGT) [14]. This theory is valid in higher dimensions as well. It is notable that the mass terms in dRGT theory are produced by consideration of a reference metric. The stability of this massive theory was studied and it was shown that this theory enjoys the absence of Boulware–Deser ghost [15]. Black hole solutions and their thermodynamical properties with considering dRGT massive gravity have been investigated in Refs. [16–18]. From the perspective of astrophysics, Katsuragawa et al. in Ref. [19], studied the neutron star in the context of this theory and showed that, the massive gravity leads to small deviation from the GR. In the cosmological context, phantom crossing and quintessence limit [20], bounce and cyclic cosmology [21], cosmological behavior [22], and other properties of this gravity have been studied by some authors [23–25].

Modification in the reference metric in dRGT theory provides the possibility of introduction of different classes of dRGT like massive theories. Among them, one can point out the one introduced by Vegh which has applications in gauge/gravity duality [26]. This theory is similar to dRGT theory with a difference that its reference metric is a singular one. Considering this theory of massive gravity, Vegh in Ref. [26], showed that graviton may behave like a lattice and exhibits a Drude peak. Also, it was pointed out that for arbitrary singular metric, this theory of massive gravity is ghost-free and stable [27]. Using this massive theory of gravity, different classes of the charged black hole solutions have been studied in Refs. [28–31]. In addition, the existence of van der Waals like behavior in extended phase space has been investigated in Refs. [28,32–35]. Holographic superconductivity in this model of massive gravity has been explored in Refs. [36–38]. Moreover, magnetic solutions of such theory have been addressed in Ref. [39]. From the astrophysical point of view, the hydrostatic equilibrium equation of the neutron stars by considering this theory of massive gravity was obtained in Ref. [40], and it was shown that the maximum mass of neutron stars can be about $3.8M_\odot$ (where $M_\odot$ is mass of the Sun).

Among the other achievements of the massive theory of gravity, one can point out the following ones: i) The cosmological constant could be realized by massive terms without the need of introduction of cosmological constant into the action [41–43]. ii) Addressing the acceleration expansion of the universe without the cosmological constant and through the properties of massive gravity [44]. iii) In large scale, the effects of massive gravity could allow the universe to accelerate while in the small scale, the effects are not on a significant level and GR is dominant. This provides a better coincidence with experimental observations [45,46]. iv) The massive gravity provides additional polarization for gravitational waves which modifies its speed of propagation [47]. This indicates that there will be a modification in the production of gravitational waves during inflation as well [48,49]. v) The maximum mass of neutron stars in massive gravity can be more than $3.2M_\odot$ [40] ($3.2M_\odot$ is the maximum mass of a neutron star in GR [50]).

Here, we consider the Vegh's approach for the massive gravity. The action of d-dimensional Einstein-massive gravity with the negative cosmological constant in the presence of Maxwell source is

$$I=-\frac{1}{16\pi}\int d^dx\sqrt{-g}\left[R-2\Lambda+L(F)+m^2\sum_i^4 c_i\mathcal{U}_i(g,f)\right], \tag{1}$$

where $\Lambda=-\frac{(d-1)(d-2)}{2l^2}$ is the negative cosmological constant, R is the scalar curvature and f is a fixed symmetric tensor. In Eq. (1), c_i are constants and $\mathcal{U}_i$'s are symmetric polynomials of the eigenvalues of the $d\times d$ matrix $K^\mu_\nu=\sqrt{g^{\mu\alpha}f_{\alpha\nu}}$ which can be written as

$$\begin{aligned}
&\mathcal{U}_1=[K], \quad \mathcal{U}_2=[K]^2-\left[K^2\right],\\
&\mathcal{U}_3=[K]^3-3[K]\left[K^2\right]+2\left[K^3\right]\\
&\mathcal{U}_4=[K]^4-6\left[K^2\right][K]^2+8\left[K^3\right][K]+3\left[K^2\right]^2-6\left[K^4\right].
\end{aligned} \tag{2}$$

Here, we want to study Maxwell electrodynamics, so the function $L(F)$ is

$$L(F)=-F, \tag{3}$$

where $F=F_{\mu\nu}F^{\mu\nu}$ (in which $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$) is the electromagnetic field tensor. Also, A_μ is the gauge potential. Variation of the action (1) with respect to the metric tensor ($g_{\mu\nu}$) and the electromagnetic field tensor ($F_{\mu\nu}$), lead to

$$G_{\mu\nu}+\Lambda g_{\mu\nu}+\frac{1}{2}g_{\mu\nu}F-2F_{\mu\lambda}F^\lambda_\nu+m^2X_{\mu\nu}=0, \tag{4}$$

$$\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right)=0, \tag{5}$$

where $G_{\mu\nu}$ is the Einstein tensor and $X_{\mu\nu}$ is the massive term with the following form

$$\begin{aligned}
X_{\mu\nu}=&-\frac{c_1}{2}\left(\mathcal{U}_1g_{\mu\nu}-K_{\mu\nu}\right)-\frac{c_2}{2}\left(\mathcal{U}_2g_{\mu\nu}-2\mathcal{U}_1K_{\mu\nu}+2K^2_{\mu\nu}\right)\\
&-\frac{c_3}{2}\left(\mathcal{U}_3g_{\mu\nu}-3\mathcal{U}_2K_{\mu\nu}+6\mathcal{U}_1K^2_{\mu\nu}-6K^3_{\mu\nu}\right)\\
&-\frac{c_4}{2}\Big(\mathcal{U}_4g_{\mu\nu}-4\mathcal{U}_3K_{\mu\nu}+12\mathcal{U}_2K^2_{\mu\nu}\\
&-24\mathcal{U}_1K^3_{\mu\nu}+24K^4_{\mu\nu}\Big).
\end{aligned} \tag{6}$$

Black hole thermodynamics has been studied widely and intensively for a long time ever since the seminal work done by Hawking et al. [51]. The amazing discovery of the thermodynamical properties of black holes helped us to have a deeper understanding of gravity and realize that the gravitational systems have some profound relations to thermodynamical systems. The concept of extended phase space was proposed [52,53] by regarding the cosmological constant Λ as thermodynamic pressure P and its conjugate quantity as thermodynamic volume V, as

$$\begin{aligned}
&M=H=U+PV, \quad P=-\frac{\Lambda}{8\pi}=\frac{3}{8\pi\ell^2}, \quad V=\left.\frac{\partial M}{\partial P}\right|_{S,Q},\\
&\Phi=\left.\frac{\partial M}{\partial Q}\right|_{S,P}, \quad S=\int_0^{r_+}T^{-1}\left(\frac{\partial M}{\partial r}\right)_{Q,P}dr,
\end{aligned} \tag{7}$$

where Q is the electric charge and Φ is its conjugate electric potential, S and T stand for the entropy and Hawking temperature on event horizon r_+, respectively. The new first law of black hole thermodynamics in the extended phase space is written as

$$dM=TdS+VdP+\Phi dQ+\Omega dJ, \tag{8}$$