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Bending angle of light in equatorial plane of Kerr–Sen Black Hole

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ABSTRACT

We study the gravitational lensing by a Kerr–Sen Black Hole arising in heterotic string theory. A closed form expression for the bending angle of light in equatorial plane of Kerr–Sen Black Hole is derived as a function of impact parameter, spin and charge of the Black Hole. Results obtained are also compared with the corresponding cases of Kerr Black Hole in general relativity. It is observed that charge parameter behaves qualitatively similar as the spin parameter for photons travelling in direct orbits while behaves differently for photons in retrograde orbits around Black Hole. As the numerical value of the Black Hole charge increases, bending angle becomes larger in strong field limit. Further it is observed that this effect is more pronounced in case of direct orbits in comparison to the retro orbits. For both the direct and retro motion, the bending angle exceeds 2π, which in turn results in multiple loops and formation of relativistic images.

1. Introduction

One of the consequences of Einstein's general relativity (GR) is that the light rays passing a massive body are deflected by the virtue of gravity and the resulting phenomenon is known as Gravitational lensing (GL) [1] as first observed by Eddington during the solar eclipse of 1919. The GL, theory and observations, is one of the most important areas in modern astronomy [2] and it also provides a clean and unique probe of the dark matter at all the distance scales since it is independent of the nature and physical state of the lensing mass [3].

In fact, the existence of most compact and extreme objects in our universe such as Black Holes (BHs) and neutron stars is now well studied in view of different independent astrophysical observations. The BHs are indeed the most fascinating objects predicted by GR [4] and in addition to the BHs in GR, there are other such Black Hole (BH) solutions in various alternative theories of gravity viz. scalar-tensor theory [5], string theory [6], braneworld scenario[7] and loop quantum gravity [8]. In particular, most of the BHs emerging in string theory [9,10] which unifies the gravity with other three fundamental forces in nature are characterized by one or more charges associated with Yang–Mills fields. Such stringy BHs may therefore provide much deeper insight into the various properties of BH spacetimes [9,10] than those of GR. The GL by a Schwarzschild BH and a Kerr BH (KBH) in the strong field limit is presented respectively in [11] and [12] in greater detail by restricting the observers in the equatorial plane. An explicit spin-dependent expression for the deflection angle in the equatorial plane of KBH is also presented in with a comparison for the case of the zero-spin BH i.e. the Schwarzschild BH in GR [13]. The detailed theoretical aspects of GL by spherically symmetric BHs in view of the perspectives for realistic observations are reviewed in [14].

More recently, the closed form expression for the deflection angle of light due to a KBH is studied with a new method under the class of asymptotic approximants [15]. This method has been successful in the description of various physical processes like thermodynamic phase behaviour [16–18] and the solution of nonlinear boundary value problems [19]. The GL in the Kerr–Sen BH (KSBH) which arises in the low energy limit of string theory [22] as a dilaton-axion generalization of the well-known KBH in GR is also performed in weak as well as strong field limits [20,21]. The KSBH has the physical properties similar to the BHs arising in Einstein–Maxwell theory, but still those can be distinguished in several aspects [22,24]. However, a careful investigation to have a closed-form expression for the bending angle of light as a function of BH spin and charge is still needed.

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The main objective of this paper is to study the GL by a Kerr–Sen BH in equatorial plane to have exact closed-form solutions for the deflection angle of light such that both the strong and weak field limits are satisfied [15–19]. In the present work, we have followed the approach used in [13], hence the results can be interpreted as the explicit generalisations of the results obtained in [13] for KBH. The main difference between our approach and the work done in [20,21] is that we have obtained an explicit expression for the bending angle for both the cases i.e. direct and retrograde motion. The final expression for bending angle depends on BH mass and spin (i.e. angular momentum per unit mass of the BH) parameters.

In present article, the exact deflection angle is derived not only in terms of impact parameter as in Schwarzschild BH case rather in terms of several external parameters viz. (BH mass, spin parameters). Similar approach for the study of the effect of the presence of plasma on gravitational lensing and relativistic images formed by Schwarzschild BH is presented in [27] and [28] in detail. The organisation of this paper is as follows. The structure of KSBH spacetime is discussed in brief in section 2 and the critical parameters in obtaining the exact deflection angles for null geodesics are then calculated in section 3. The closed-form expression for the deflection angle as a function of impact parameter and BH spin is derived in section 4. Finally, the results obtained are concluded in section 5 along with the future directions.

2. Kerr–Sen BH spacetime

The KSBH spacetime is described by the following 4D effective action [22],

$$S = -\int d^4x\sqrt{-\mathcal{G}}e^{-\Phi}\left(-\mathcal{R} + \frac{1}{12}\mathcal{H}^2 - \mathcal{G}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{8}\mathcal{F}^2\right), \tag{1}$$

where Φ is the dilaton field and $\mathcal{R}$ is the scalar curvature, $\mathcal{F}^2 = \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$ with the field strength $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$ which corresponds to the Maxwell field $\mathcal{A}_\mu$, and $\mathcal{H}^2 = \mathcal{H}_{\mu\nu\rho}\mathcal{H}^{\mu\nu\rho}$ with $\mathcal{H}_{\mu\nu\rho}$ given by

$$\mathcal{H}_{\mu\nu\rho} = \partial_\mu\mathcal{B}_{\nu\rho} + \partial_\nu\mathcal{B}_{\rho\mu} + \partial_\rho\mathcal{B}_{\mu\nu} - \frac{1}{4}\left(\mathcal{A}_\mu\mathcal{F}_{\nu\rho} + \mathcal{A}_\nu\mathcal{F}_{\rho\mu} + \mathcal{A}_\rho\mathcal{F}_{\mu\nu}\right), \tag{2}$$

where the last term in Eq. (2) is the gauge Chern–Simons term however $\mathcal{G}_{\mu\nu}$ as appeared in Eq. (1) are the covariant components of the metric in the string frame, which are related to the Einstein metric by $g_{\mu\nu} = e^{-\Phi}\mathcal{G}_{\mu\nu}$. The Einstein metric for KSBH, the non-vanishing components of A_μ, $B_{\mu\nu}$ and the dilaton field respectively read as below [22],

$$ds^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 - \frac{4\mu ar\cosh^2\alpha\sin^2\theta}{\Sigma}dtd\phi + \Sigma d\theta^2 + \frac{\Xi\sin^2\theta}{\Sigma}d\phi^2, \tag{3}$$

$$\mathcal{A}_t = \frac{\mu r\sinh 2\alpha}{\sqrt{2}\Sigma}, \quad \mathcal{A}_\phi = \frac{\mu\, a\, r\sinh 2\alpha\sin^2\theta}{\sqrt{2}\Sigma}, \tag{4}$$

$$\mathcal{B}_{t\phi} = \frac{2a^2\mu r\sin^2\theta\sinh^2\alpha}{\Sigma}, \quad \Phi = -\frac{1}{2}\ln\frac{\Sigma}{r^2 + a^2\cos^2\theta}, \tag{5}$$

where the metric functions are described as,

$$\Delta = r^2 - 2\mu r + a^2, \tag{6}$$

$$\Sigma = r^2 + a^2\cos^2\theta + 2\mu r\sinh^2\alpha, \tag{7}$$

$$\Xi = \left(r^2 + 2\mu r\sinh^2\alpha + a^2\right)^2 - a^2\Delta\sin^2\theta. \tag{8}$$

The parameters μ, α and a are related to the physical mass M, charge Q and angular momentum J as follows,

$$M = \frac{\mu}{2}(1 + \cosh 2\alpha), \qquad Q = \frac{\mu}{\sqrt{2}}\sinh^2 2\alpha, \qquad J = \frac{a\mu}{2}(1 + \cosh 2\alpha). \tag{9}$$

Solving Eq. (9), one can obtain,

$$\sinh^2\alpha = \frac{Q^2}{2M^2 - Q^2}, \qquad \mu = M - \frac{Q^2}{2M}. \tag{10}$$

Then the parameters α and μ in the metric (3) can be eliminated accordingly. For a nonextremal BH, there exist two horizons, determined by $\Delta(r) = 0$ as,

$$r_\pm = M - \frac{Q^2}{2M} \pm \sqrt{\left(M - \frac{Q^2}{2M}\right)^2 - a^2}, \tag{11}$$

where r_+ and r_- represent the outer and the inner horizons of the BH respectively. The case of extremal KSBH requires,

$$Q^2 = 2M(M - a). \tag{12}$$