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# Holographic entanglement entropy and entanglement thermodynamics of 'black' non-susy D3 brane

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ABSTRACT

Like BPS D3 brane, the non-supersymmetric (non-susy) D3 brane of type IIB string theory is also known to have a decoupling limit and leads to a non-supersymmetric AdS/CFT correspondence. The throat geometry in this case represents a QFT which is neither conformal nor supersymmetric. The 'black' version of the non-susy D3 brane in the decoupling limit describes a QFT at finite temperature. Here we first compute the entanglement entropy for small subsystem of such QFT from the decoupled geometry of 'black' non-susy D3 brane using holographic technique. Then we study the entanglement thermodynamics for the weakly excited states of this QFT from the asymptotically AdS geometry of the decoupled 'black' non-susy D3 brane. We observe that for small subsystem this background indeed satisfies a first law like relation with a universal (entanglement) temperature inversely proportional to the size of the subsystem and an (entanglement) pressure normal to the entangling surface. Finally we show how the entanglement entropy makes a cross-over to the thermal entropy at high temperature.



## 1. Introduction

The entanglement entropy (EE) is a measure of quantum information encoded in a quantum system. In particular, for a bipartite system the EE of a subsystem $A$ is the von Neumann entropy and is defined as $S_A = -\text{Tr}(\rho_A \log \rho_A)$, where $\rho_A = \text{Tr}_B(\rho_{\text{tot}})$ is the reduced density matrix on $A$ obtained by tracing out on $B$, the complement of $A$, of the density matrix of the total system $\rho_{\text{tot}}$ (see, for example, [1–7] including some reviews). It is useful for many body systems to describe various quantum phases of matter and serves as an order parameter for the quantum phase transitions which occur near zero temperature [8–12]. The density matrix can be carefully defined in the continuum and therefore, EE can be calculated in a QFT in principle using the so-called replica trick (see, for example [13]). However, the actual computation can be done quite generally only in low dimensional $\text{CFT}_{d+1}$ ($d < 2$) [3,4]. For higher dimensions the computation of EE becomes intractable except for some special cases, like free field QFT in three dimensions and also for $\text{CFT}_4$ [13].

Ryu and Takayanagi [14,15], motivated by the Bekenstein–Hawking entropy formula, gave a prescription to compute EE in any dimensions using the idea of AdS/CFT [16,17]. According to them, the holographic EE (HEE) of the subsystem $A$ in the gravity dual is given by [14]

$$S_E = \frac{\text{Area}(\gamma_A^{\text{min}})}{4G_N} \tag{1}$$

where $\gamma_A^{\text{min}}$ is the $d$-dimensional minimal area (time-sliced) surface in $\text{AdS}_{d+2}$ space whose boundary matches with the boundary of the subsystem $A$, i.e., $\partial\gamma_A^{\text{min}} = \partial A$ and $G_N$ is the $(d+2)$-dimensional Newton's constant. The HEE given in (1) has been checked [14] to agree with the QFT results in lower dimensions. In higher dimensions also they give correct qualitative behaviors. In thermodynamics the entropy of a system can be increased by injecting energy to the system, where the proportionality constant is given by the inverse of temperature. This leads to an energy conservation relation $\Delta E = T\Delta S$, the first law of thermodynamics. An analogous problem was addressed in [18] for the EE, i.e., to see how the EE of a certain region grows with the increase in energy. Here the EE is computed using AdS/CFT. The excited state of a CFT is given by the deformation of AdS whose EE can be computed using (1). This is then compared with the time component of the boundary stress tensor $T_{tt}$ or the energy density. For a small subsystem $A$, the total energy is found to be proportional to the increase in EE and the proportionality constant is $c/\ell$,

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where $c$ is a universal constant and $\ell$ is the size of the subsystem. This has been identified with the entanglement temperature in analogy with first law of thermodynamics [18]. However, in [19], it has been noted that this is not the complete story. Since the first law contains more terms here also $\Delta E$ can have a term analogous to $P\Delta V$ term. Indeed, by calculating the other components of the boundary stress tensor it has been found that $\Delta E$ contains a term $d/(d+2)V_d\Delta P_x$ for asymptotically AdS$_{d+2}$ space, where $\Delta P_x$ is the pressure normal to the entangling surface and $V_d$ is the volume. Therefore the analogous entanglement thermodynamical relation takes the form [19],

$$\Delta E = T_E \Delta S_E + \frac{d}{d+2} V_d \Delta P_x \tag{2}$$

In this paper we consider the non-susy D3 brane or, to be precise, a finite temperature version of that solution in type IIB string theory [20]. It is known that like BPS D3 brane, non-susy D3 brane also has a decoupling limit [21,22] and therefore, gives a gravity dual of a non-supersymmetric finite temperature gauge theory in the decoupling limit. The gauge theory in this case is non-conformal. We use this gravity dual to compute the EE of the associated QFT from the Ryu–Takayanagi prescription (1). Since the non-susy D3 brane in the decoupling limit has an asymptotically AdS$_5$ geometry, the HEE can be written as a pure AdS$_5$ part and additional part which can be thought of as the EE associated with an excited state. We use Fefferman–Graham coordinate to compute the HEE and this helps us to identify the boundary stress tensor quite easily [23,24]. Having identified the boundary stress tensor we then check that the additional EE of the excited state indeed satisfies the first law like thermodynamical relation we just mentioned in (2) for small subsystem. We have identified the entanglement temperature in this case which is inversely related to the size of the entangling region by a universal constant and also an entanglement pressure normal to the entangling surface. Although non-susy D3 brane we are considering here has a naked singularity, one can define a temperature related to one of the parameters of the solution. When the parameter takes a particular value the solution reduces to the standard Schwarzschild AdS$_5$ solution and we checked that for that particular value of the parameter our results reduce to those obtained in earlier works [18]. We also checked that at higher temperature the HEE makes a cross-over [25] to the thermal entropy of standard black D3 brane.

The rest of the paper is organized as follows. In section 2, we briefly discuss the decoupled geometry of 'black' non-susy D3 brane solution. The Fefferman–Graham coordinate and the computation of HEE is given in section 3. In section 4, we give the boundary stress tensors and study the entanglement thermodynamics. The cross-over of the HEE to Bekenstein–Hawking thermal entropy is discussed in section 5. Finally we conclude in section 6.

## 2. Decoupled geometry of 'black' non-susy D3 brane

The 'black' non-susy D3 brane solution of type IIB string theory has been discussed in detail in [20] and so we will be brief here. The purpose for our discussion here is to fix the notation and convention for the computation of HEE in the next section. The solution in the Einstein frame takes the form,

$$ds^2 = F_1(\rho)^{-\frac{1}{2}} G(\rho)^{-\frac{\delta_2}{8}} \left[ -G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + F_1(\rho)^{\frac{1}{2}} G(\rho)^{\frac{1}{4}} \left[ \frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2 \right]$$

$$e^{2\phi} = G(\rho)^{-\frac{3\delta_2}{2}+\frac{7\delta_1}{4}}, \qquad F_{[5]} = \frac{1}{\sqrt{2}}(1+*)Q\,\mathrm{Vol}(\Omega_5), \tag{3}$$

where the functions $G(\rho)$ and $F(\rho)$ are defined as,

$$G(\rho) = 1 + \frac{\rho_0^4}{\rho^4},$$
$$F_1(\rho) = G(\rho)^{\frac{\alpha_1}{2}} \cosh^2\theta - G(\rho)^{-\frac{\beta_1}{2}} \sinh^2\theta \tag{4}$$

Here $\delta_1$, $\delta_2$, $\alpha_1$, $\beta_1$, $\theta$, $\rho_0$, $Q$ are the parameters characterizing the solution. Now to compare this solution with that given in eq. (6) of [20], we note that we have replaced $\delta$ by $\delta_2$ here and also, the function $F(\rho)$ there is related to $F_1(\rho)$ by the relation $F_1(\rho) = G(\rho)^{3\delta_1/8}F(\rho)$. The parameters $\alpha$ and $\beta$ there are related to $\alpha_1$ and $\beta_1$ by the relations $\alpha_1 = \alpha + 3\delta_1/4$ and $\beta_1 = \beta - 3\delta_1/4$. We point out that the parameters are not all independent but they satisfy the following relations

$$\alpha_1 - \beta_1 = \alpha - \beta + 3\delta_1/2 = 0$$
$$\alpha_1 + \beta_1 = \alpha + \beta = \sqrt{10 - \frac{21}{2}\delta_2^2 - \frac{49}{2}\delta_1^2 + 21\delta_2\delta_1}$$
$$Q = (\alpha_1 + \beta_1)\rho_0^4 \sinh 2\theta \tag{5}$$

Note that the solution has a curvature singularity at $\rho = 0$ and also the metric does not have the full Poincare symmetry ISO(1, 3) in the brane world-volume directions, rather, it is broken to R $\times$ ISO(3) and this is the reason we call it 'black' non-susy D3 brane solution. However, we put black in inverted comma because this solution does not have a regular horizon as in ordinary black brane but, has a singular horizon. The standard zero temperature non-susy D3 brane solution given in eq. (1) of [22] can be recovered from (3) by simply putting $\delta_2 = 0$ and identifying $7\delta_1/4$ as $\delta$ there. We remark that in spite of the solution (3) has a singular horizon we can still define a temperature as argued in [26] and by comparing the expression for temperature there we can obtain the temperature of the 'black' non-susy D3 brane as,

$$T_{\text{nonsusy}} = \left( \frac{-2\delta_2}{(\alpha_1+\beta_1)^2} \right)^{\frac{1}{4}} \frac{1}{\pi\rho_0 \cosh\theta} \tag{6}$$

From the above expression it is clear that for the reality of the temperature the parameter $\delta_2$ must be less or equal to zero. It is straightforward to check that when $\delta_2 = -2$ and $\delta_1 = -12/7$ (which implies $\alpha_1 = \beta_1 = 1$ and $\alpha_1 + \beta_1 = 2$), the above solution (3) reduces precisely to the ordinary black D3 brane solution and the temperature (6) also reduces to the Hawking temperature of the ordinary black D3 brane.

From now on we will put $\alpha_1 + \beta_1 = 2$ for simplicity. Therefore, from the first relation in (5), we have $\alpha_1 = 1$ and $\beta_1 = 1$. In this case, the parameters $\delta_1$ and $\delta_2$ will be related (see the second equation in (5)) by

$$42\delta_2^2 + 49\delta_1^2 - 84\delta_1\delta_2 = 24 \tag{7}$$

The function $F_1(\rho)$ given in (4) then reduces to

$$F_1(\rho) = G(\rho)^{-\frac{1}{2}} H(\rho), \qquad \text{where,}$$
$$H(\rho) = 1 + \frac{\rho_0^4 \cosh^2\theta}{\rho^4} \equiv 1 + \frac{\rho_1^4}{\rho^4} \tag{8}$$

Therefore the Einstein frame metric in (3) reduces to

$$ds^2 = H(\rho)^{-\frac{1}{2}} G(\rho)^{\frac{1}{4}-\frac{\delta_2}{8}} \left[ -G(\rho)^{\frac{\delta_2}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + H(\rho)^{\frac{1}{2}} \left[ \frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2 \right] \tag{9}$$