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Black hole horizons can hide positive heat capacity

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ABSTRACT

Regarding the volume as independent thermodynamic variable we point out that black hole horizons can hide positive heat capacity and specific heat. Such horizons are mechanically marginal, but thermally stable. In the absence of a canonical volume definition, we consider various suggestions scaling differently with the horizon radius. Assuming Euler-homogeneity of the entropy, besides the Hawking temperature, a pressure and a corresponding work term render the equation of state at the horizon thermally stable for any meaningful volume concept that scales larger than the horizon area. When considering also a Stefan–Boltzmann radiation like equation of state at the horizon, only one possible solution emerges: the Christodoulou–Rovelli volume, scaling as $V \sim R^5$, with an entropy $S = \frac{8}{3} S_{BH}$.

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1. Introduction

The irreducible mass of black holes is connected to an entropy function in black hole thermodynamics [1–5]. This relation inspired many further investigations about the origin of the fundamental equations including various ideas toward quantum gravity [6–14]. It is well known that the related equation of state has some peculiar properties from a thermodynamic point of view. Due to the fact that the irreducible mass of black holes is proportional to the radius of their event horizon, the entropy, proportional to its surface, $S(M) \sim M^2$, is seemingly convex and the heat capacity derived from it is negative. This is common in all bound systems where the total energy is negative and the kinetic energy is positive, then due to an increase of the temperature via an increase in the kinetic energy, – in a stationary state satisfying a virial theorem, – the total energy will decrease, displaying formally a negative heat capacity. A thermal equilibrium between a negative specific heat system and a positive one is, however, not possible. Black holes in this sense seem thermally unstable.

There are various suggestions that could counterbalance the consequent mechanical instability [15–18], however, its very existence is an obstacle in constructing reasonable statistical theories for black holes [19–22]. A careful distinction of extensivity and additivity in the related thermostatistics promises to give an insight into the problem [23,24], and a Rényi entropy [25] based theory actually removes the convexity of the Bekenstein–Hawking entropy of black holes [26–28].

In this Letter we demonstrate that the black hole horizon entropy formula is concave if treated as a function of at least two variables, and leads to "normal" thermodynamic behavior, with positive specific heat and marginal mechanical stability. We argue that considering any reasonable volume concepts (e.g. the Parikh [29] or the Christodoulou–Rovelli definition [30]) as an independent thermodynamical variable together with the related homogeneity assumption, eliminates the inconsistency while keeping the original formula.

First a brief review of the custom derivation is given which leads to the currently accepted conclusion of assigning negative heat capacity to such objects. Then we derive the thermodynamic properties of Schwarzschild black holes by including the usual work term in the first law based only on the assumption that the entropy is a first order homogeneous (extensive) function of the volume. Throughout this work we use units such as $\hbar = G = c = k_B = 1$.

2. Black hole EoS with volume term

The traditional presentation of the negative heat capacity problem is as follows: Schwarzschild black hole horizons have a radius of $R = 2M$, and a Bekenstein–Hawking entropy of a quarter of the horizon area

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$$S = \pi R^2. \tag{1}$$

Since the internal energy is dominated by the mass energy producing the same horizon, $E = M = R/2$, one light-heartedly considers a curious equation of state:

$$S(E) = 4\pi\, E^2. \tag{2}$$

This "equation of state" has strange properties. The absolute temperature, determined from

$$\frac{1}{T} = \frac{dS}{dE} = 8\pi\, E, \tag{3}$$

is growing with decreasing energy. This discrepancy results a negative heat capacity signaling *thermal instability* in the traditional view:

$$-\frac{1}{CT^2} = \frac{d^2S}{dE^2} = 8\pi > 0, \tag{4}$$

which leads to the conclusion of having $C = -2S < 0$. Negative heat capacity occurs in all systems having negative total energy. It is questionable, however, whether the total energy has to be counted as internal energy when deriving thermal properties of a system.

Here we present an alternative approach which is thermodynamically consistent, and free from such oddities. First of all we consider the volume, enclosed by the event horizon, as a further thermodynamical variable. The physical volume of a black hole has been a long standing problem in general relativity. The standard definition operates with surfaces of simultaneity and therefore it is a strongly coordinate dependent notion. Recently, Christodoulou and Rovelli introduced an elegant, geometric invariant definition [30], where the volume of a Schwarzschild black hole has been defined as the largest, spherically symmetric, spacelike hypersurface Σ bounded by the horizon. The corresponding CR-volume (when the thermal property of the Hawking radiation [31] is also taken into account) scales as $V \sim R^5$, which for an astrophysical black hole turns out to be very large indeed. This result motivated further investigations about the role this volume may play in the thermodynamic behavior of black holes [32–36], in particular, based only on simple causality considerations, Rovelli argues [37] that black holes should have more states than those giving the Bekenstein–Hawking entropy, and the CR-volume is large enough to store these entropic states.

In this Letter we consider the phenomenological consequences of the volume scaling of the black hole entropy, however we do not restrict our investigations to the CR-measure only. The approach taken here is completely general and valid for any meaningful volume definition. We will show, however, that by considering a Stefan–Boltzmann radiation like equation of state at the horizon (arising naturally from a Hawking radiation), the CR-volume scaling is reproduced.

The step to consider the volume as a thermodynamic variable is a fundamental one which also associates a pressure to the event horizon. In standard thermodynamics there exists a relationship (the Gibbs–Duhem relation (see e.g. [38])) among the intensive parameters of a system which is a consequence of the first order homogeneous property of the entropy function. This homogeneity relation is not valid within the standard picture of black hole mechanics (see e.g. [23,39] and references therein). A modification by York and Martinez [40–42] tries to separate the surface of the horizon as an independent thermodynamic variable, however, the consequent scaling relations are not first order Euler-homogeneous, therefore there is no real Gibbs–Duhem relation in that framework [43].

In the present approach, separating the volume to be the independent thermodynamic variable naturally resolves the Gibbs–Duhem relation issue, which, together with the well-known power-law scaling of the energy, E, the total entropy, S, and the volume, V, with the horizon radius, R of a Schwarzschild black hole, naturally suggests the general class of equation of states in the form

$$S(E, V) = \zeta\, E^{\alpha}\, V^{\beta}, \tag{5}$$

and the Euler-homogeneity assumption sets the condition $\alpha+\beta=1$. This form of equation of state does not contradict to the "no hair" theorem [44], as long as both $E(M)$ and $V(M)$ depend only on the sole physically relevant property of a Schwarzschild black hole, its mass M. Nevertheless $S(E, V)$ has to be handled as a two-variable function when obtaining its partial derivatives, and their corresponding physical interpretation. Only these have to be taken at the end on physical line described by the pair $(E(M), V(M))$ in the parameter space. Temperature, partial derivative against E, is no more or less physical than pressure, obtained from partial derivative against V. Microscopically both the absolute temperature and pressure are positive in kinetic theories, while the classical pressure may turn out to be negative in bound systems. In those cases the quantum uncertainty may stabilize such systems. But this very same actor is responsible for the Unruh-type Hawking temperature.

In order to further specify the black hole equation of state by keeping the power-law form and without the loss of generality, one can parametrize the volume as

$$V = R^{c+3}\, I_V, \tag{6}$$

where I_V is constant, independent of the horizon radius. For any choice of $c \neq 0$ the volume in the present context is not the Euclidean three-volume, usually considered in everyday thermodynamics. According to the Schwarzschild black hole picture, the required dependence of the total energy on the radius, $E = M = R/2$, and the total entropy is proportional to the horizon area, $S = 4\pi\lambda R^2 = \pi\lambda M^2$, where $\lambda = 1/4$ for the Bekenstein–Hawking entropy.

The scaling of the volume with the radius, i.e. the parameter c, remains undetermined so far. The parameter λ together with I_V stays also undetermined at this level. From the equation of state (5) we have

$$4\pi\lambda R^2 = \zeta\, (R/2)^{\alpha}\, (I_V R^{c+3})^{\beta} \tag{7}$$

and therefore

$$2 = \alpha + \beta(c+3). \tag{8}$$

For further specification of the parameters we need more input from the physical picture. Calculating the thermodynamical derivatives of $S(E, V)$ one interprets the temperature

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \alpha\zeta E^{\alpha-1} V^{\beta} = \alpha\frac{S}{E} = 8\pi\lambda\alpha R. \tag{9}$$

This temperature T is to be equal to the Hawking temperature [31], $T_H = 1/(4\pi R)$, which is the Unruh temperature [45], belonging to the gravitational acceleration at the horizon (without the red-shift factor). Keeping this equality delivers $\lambda = 1/(2\alpha)$. The other partial derivative,

$$\frac{p}{T} = \frac{\partial S}{\partial V} = \beta\zeta E^{\alpha} V^{\beta-1} = \beta\frac{S}{V}, \tag{10}$$

leads to another form of the equation of state, that is generally more useful in hydrodynamical calculations,

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