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Exact solution of mean-field plus an extended T = 1 nuclear pairing Hamiltonian in the seniority-zero symmetric subspace

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ABSTRACT

An extended pairing Hamiltonian that describes multi-pair interactions among isospin $T = 1$ and angular momentum $J = 0$ neutron–neutron, proton–proton, and neutron–proton pairs in a spherical mean field, such as the spherical shell model, is proposed based on the standard $T = 1$ pairing formalism. The advantage of the model lies in the fact that numerical solutions within the seniority-zero symmetric subspace can be obtained more easily and with less computational time than those calculated from the mean-field plus standard $T = 1$ pairing model. Thus, large-scale calculations within the seniority-zero symmetric subspace of the model is feasible. As an example of the application, the average neutron–proton interaction in even–even $N \sim Z$ nuclei that can be suitably described in the f_5pg_9 shell is estimated in the present model, with a focus on the role of np-pairing correlations.

1. Introduction

The pairing interaction is known to be very important for mean-field descriptions of ground-state and low-energy properties of nuclei [1,2]. It has been shown that either spherical or deformed mean-field plus the standard (orbit-independent) pairing interaction among angular momentum $J = 0$ like-nucleon pairs can be solved exactly by using the Gaudin–Richardson method [3–5]. The deformed and spherical mean-field plus the extended pairing interaction among $J = 0$ like-nucleon pairs have also been proposed, which can be solved more easily than the standard pairing model, especially when both the number of like-nucleon pairs and the number of single-particle orbits are large [6,7]. It is also known that the ground-state properties and some properties of low-lying states of a chain of isotopes or isotones can be well described by these exactly solvable models [6–12]. Furthermore, as shown in [13], the extended multi-pairing interaction among like-nucleon pairs [6] can be obtained from the standard pairing interaction with an approximation, in which only the lowest eigenstate and the eigen-energy of the standard pairing interaction are taken into account. Actually, as shown in [13], this part of the standard pairing interaction, expressed as the extended multi-pairing interaction form, plays a dominant role for low-lying states, while the remaining part of the standard pairing interaction is less important to the low-lying states, especially when the number of nucleon pairs is small, which elucidates the origin of the extended pairing interaction. Hence, properties of low-lying states described by the extended pairing model are essentially the same as those described by the standard pairing model.

Extensions to equal strength neutron–neutron (nn), proton–proton (pp), and neutron–proton (np) isospin $T = 1$ (charge-independent) pairing interactions has also been formulated [14–18], in which the total isospin T is a conserved quantity. Specifically, it has been shown that the $T = 1$ pairing Hamiltonian, which will be called the standard $T = 1$ pairing in the following, can be built from generators of the quasi-spin $O(5)$ group. However, a practical algorithm for diagonalizing a model with the $T = 1$ pairing interaction in coupled or uncoupled basis of $O(5)$ irreducible representations (irreps) is still lacking. It should also be stated that, similar to the pairing model for like-nucleon pairs, approximate numerical solutions of the mean-field plus standard $T = 1$ pairing Hamiltonian can also be obtained by using the BCS or HFB formalism [19–21], while simplified but reasonable exact solutions can be achieved by using an average energy (centroid) of the p orbits (e.g., see [22] for the simplest seniority-zero case). Exact solution of the mean-field plus standard $T = 1$ pairing model was considered previously [23,24]. The common feature lies in the fact that a set of coupled multi-variable polynomial equations are involved, in which the order of the polynomials increases with in-

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creasing number of orbits and total number of nucleon-pairs as demonstrated in [25] for applications of [23] for up to three nucleon pairs around the cores of ^{16}O, ^{40}Ca, and ^{56}Ni. Though there is no practical limitations for the application of the exact solution of the standard T = 1 pairing in nuclei, it will be helpful if there is a reasonably simplified model to the problem that can be solved more easily.

2. An extended $T = 1$ pairing model and its exact solution

For a p-orbit system, the standard $T = 1$ pairing Hamiltonian is given by

$$\hat{H}_{\rm SP} = -G \sum_{\mu} A^{\dagger}_{\mu} A_{\mu}, \tag{1}$$

where $G > 0$ is the overall pairing interaction strength,

$$A^{\dagger}_{\mu} = \sum_{i=1}^{p} A^{\dagger}_{\mu}(j_i) = \sum_{i=1}^{p} \sum_{m_i>0} (-)^{j_i - m_i} a^{\dagger}_{j_i, m_i, \mu/2} a^{\dagger}_{j_i, -m_i, \mu/2}$$
for $\mu = 1$ or -1,
$$A^{\dagger}_{0} = \sum_{i=1}^{p} A^{\dagger}_{0}(j_i) = \sqrt{\frac{1}{2}} \sum_{i=1}^{p} \sum_{m_i>0} (-)^{j_i - m_i} (a^{\dagger}_{j_i, m_i, 1/2} a^{\dagger}_{j_i, -m_i, -1/2} + a^{\dagger}_{j_i, m_i, -1/2} a^{\dagger}_{j_i, -m_i, 1/2}) \tag{2}$$

are nucleon-pair creation operators, in which $a^{\dagger}_{j_i, m_i, m_t}$ (a_{j_i, m_i, m_t}) is the creation (annihilation) operator for a nucleon in the i-th orbit of a mean-field with angular momentum j_i, angular momentum projection m_i, and isospin projection m_t with $m_t = 1/2$ or $-1/2$. As shown in [14–17], $\{A^{\dagger}_{\mu}, A_{\mu}\}$, together with the number operator of total nucleons $\hat{N} = \sum_{i=1}^{p} \hat{N}_{j_i}$ and the isospin operators $T_{\mu} = \sum_{i=1}^{p} T_{\mu}(j_i)$ ($\mu = +, -, 0$), generate the quasi-spin $O(5)$ algebra, of which the commutation relations can be found, for example, in [23].

Let $|\rho\rangle$ be the orthonormalized basis vectors of $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$, in which $\rho \equiv \{(\omega_1, \omega_2)\, \beta\, \mathcal{N}\, T\, M_T;\ \eta\}$, where $(\omega_1, \omega_2) = (\Omega - v/2, t)$ is an irrep of $O(5)$ occurring in the reduction of the Kronecker product of p copies of $O(5)$ irreps $\otimes_{i=1}^{p} (\omega_{1,i}, \omega_{2,i})$ of $O_1(5) \otimes \cdots \otimes O_p(5) \downarrow O(5)$, $\Omega = \sum_{i=1}^{p} \Omega_i = \sum_i (j_i + 1/2)$, v is the total seniority number, t is the reduced isospin of unpaired nucleons, β is the branching-multiplicity label needed in the $O(5) \downarrow O_T(3) \otimes O_{\mathcal{N}}(2)$ reduction, T and M_T are quantum number of total isospin and that of its projection, respectively, $\mathcal{N} = \Omega - N/2$ with N being the total number of nucleons, and η stands for a set of other quantum numbers related to the total angular momentum. Thus, $\{|\rho\rangle\}$ is a complete set of basis vectors needed in the $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$ basis. The standard $T = 1$ pairing interaction Hamiltonian (1) can then be expressed in terms of its complete set of eigenvalues and the corresponding eigenstates as

$$\hat{H}_{\rm SP} = \sum_{\rho} E^{\rho} |\rho\rangle\langle\rho|, \tag{3}$$

where the sum runs over all possible ρ. Since the eigenstates with $v = 0$ are the lowest in eigen-energy, similar to the extended quasi-spin $SU(2)$ pairing interaction [13], only the $v = 0$ sector involved in (3) will be adopted, with the other sectors that lie higher in energy and therefore less important than the $v = 0$ sector neglected as an approximation. To do so, one may equivalently introduce a projected $T = 1$ pairing interaction with

$$\tilde{H}_{\rm SP} = P_{v=0}\, \hat{H}_{\rm SP}\, P_{v=0}, \tag{4}$$

where

$$P_{v=0} = \sum_{\mathcal{N}\, T\, M_T} |(\Omega, 0)\mathcal{N}\, T\, M_T\rangle\langle(\Omega, 0)\mathcal{N}\, T\, M_T| \tag{5}$$

is a projection operator, in which the label β can be omitted because the reduction $(\Omega, 0) \downarrow (\mathcal{N}, T)$ is branching-multiplicity-free. It can be proven directly that

$$[C_2(O(5)),\ \tilde{H}_{\rm SP}] = 0,\ [\hat{\mathcal{N}},\ \tilde{H}_{\rm SP}] = 0,$$
$$[T_{\mu},\ \tilde{H}_{\rm SP}] = 0 \ \text{for}\ \mu = +,\ -,\ 0 \tag{6}$$

still hold, so that the projected Hamiltonian (4) preserves the $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$ symmetry.

In the second quantization picture, for given Ω_i ($i = 1, \cdots, p$), the Hamiltonian (4) is

$$\tilde{H}_{\rm SP} = \sum_{n\, T\, M_T} E^{(\Omega,\,0)\, n\, T} \sum_{\rho_1, \cdots, \rho_p, \tilde{\rho}_1, \cdots, \tilde{\rho}_p} F^{n\, T\, M_T}_{\rho_1, \cdots, \rho_p} \times F^{n\, T\, M_T}_{\tilde{\rho}_1, \cdots, \tilde{\rho}_p} \prod_{i=1}^{p} K^{-1}_{n_i\, T_i} Z^{(n_i\, 0)}_{T_i\, M_{T,\, i}} [\mathbf{A}^{\dagger}(j_i)] \times \prod_{i'=1}^{p} K^{-1}_{\tilde{n}_{i'}\, \tilde{T}_{i'}} Z^{(\tilde{n}_{i'}\, 0)}_{\tilde{T}_{i'}\, \tilde{M}_{T,\, i'}} [\mathbf{A}(j_{i'})], \tag{7}$$

where $E^{(\Omega,\,0)\, n\, T} = -\frac{G_{\rm ext}}{2} (n(2\Omega + 3 - n) - T(T+1))$, in which n is the total number of nucleon-pairs, while the overall pairing strength G of the standard $T = 1$ pairing interaction is replaced by $G_{\rm ext}$, the additional quantum numbers η_i can be omitted in this case with $\rho_i \equiv \{n_i\, T_i\, M_{T,i}\}$ and $\tilde{\rho}_i \equiv \{\tilde{n}_i\, \tilde{T}_i\, \tilde{M}_{T,i}\}$, in which n_i or $\tilde{n}_i$ is the number of nucleon-pairs in the i-th orbit, $Z^{(n\, 0)}_{T M_T}[\mathbf{A}^{\dagger}]$ is the polynomial of $\{A^{\dagger}_{\mu}\}$ given by [18]

$$Z^{(n\, 0)}_{T M_T}[\mathbf{A}^{\dagger}] = \left[\frac{2^{T+M_T}(2T+1)!!(T+M_T)!(T-M_T)!T!}{(n-T)!!(n+T+1)!!(2T)!}\right]^{\frac{1}{2}} \times \left(2\, A^{\dagger}_1 A^{\dagger}_{-1} - A^{\dagger 2}_0\right)^{\frac{n-T}{2}} \times \sum_{x={\rm Max}[0, M_T]}^{[(T+M_T)/2]} \frac{A^{\dagger x}_1 A^{\dagger\, T+M_T-2x}_0 A^{\dagger\, x-M_T}_{-1}}{2^x (x-M_T)! x! (T+M_T-2x)!}, \tag{8}$$

where x should be positive integer, and $[y]$ denotes the integer part of y.

$$K^{-1}_{n\, T} = \left[\frac{2^{\frac{1}{2}(n-T)}(\Omega - (n+T)/2)!(2\Omega + 1 - n + T)!!}{\Omega_i!(2\Omega+1)!!}\right]^{\frac{1}{2}}, \tag{9}$$

and

$$F^{n\, T\, M_T}_{\rho_1, \cdots, \rho_p} = \langle \rho_1, \cdots, \rho_p | (\Omega, 0) \mathcal{N}\, T\, M_T \rangle \tag{10}$$

is the $O(5) \supset (O_T(3) \supset O_T(2)) \otimes O_{\mathcal{N}}(2)$ multi-coupling coefficient. According to the vector coherent state theory [18], in general, the overlap $F^{n\, T\, M_T}_{\rho_1, \cdots, \rho_p}$ can be expressed as