JID:PLB AID:33825 /SCO Doctopic: Theory [m5Gv1.3; v1.236; Prn:22/05/2018; 7:46] P.1 (1-11)

Physics Letters B ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

AdS and Lifshitz black hole solutions in conformal gravity sourced with a scalar field

Felipe Herrera, Yerko Vásquez

Departamento de Física y Astronomía, Facultad de Ciencias, Universidad de La Serena, Avenida Cisternas 1200, La Serena, Chile

ARTICLE INFO

Article history:
Received 10 February 2018
Received in revised form 16 May 2018
Accepted 18 May 2018
Available online xxxx
Editor: N. Lambert

ABSTRACT

In this paper we obtain exact asymptotically anti-de Sitter black hole solutions and asymptotically Lifshitz black hole solutions with dynamical exponents $z=0$ and $z=4$ of four-dimensional conformal gravity coupled with a self-interacting conformally invariant scalar field. Then, we compute their thermodynamical quantities, such as the mass, the Wald entropy and the Hawking temperature. The mass expression is obtained by using the generalized off-shell Noether potential formulation. It is found that the anti-de Sitter black holes as well as the Lifshitz black holes with $z=0$ have zero mass and zero entropy, although they have non-zero temperature. A similar behavior has been observed in previous works, where the integration constant is not associated with a conserved charge, and it can be interpreted as a kind of gravitational hair. On the other hand, the Lifshitz black holes with dynamical exponent $z=4$ have non-zero conserved charges, and the first law of black hole thermodynamics holds. Also, we analyze the horizon thermodynamics for the Lifshitz black holes with $z=4$, and we show that the first law of black hole thermodynamics arises from the field equations evaluated on the horizon. Furthermore, we study the propagation of a conformally coupled scalar field on these backgrounds and we find the quasinormal modes analytically in several cases. We find that for anti-de Sitter black holes and Lifshitz black holes with $z=4$, there is a continuous spectrum of frequencies for Dirichlet boundary condition; however, we show that discrete sets of well defined quasinormal frequencies can be obtained by considering Neumann boundary conditions.

1. Introduction

An interesting class of space–times that have received considerable attention from the point of view of condensed matter physics are the Lifshitz space–times, which are described by the metrics [1]

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}}dt^2 + \frac{\ell^2}{r^2}dr^2 + \frac{r^2}{\ell^2}d\vec{x}^2 \,, \tag{1.1}$$

where $\vec{x}$ represents a $D-2$ dimensional spatial vector, with D corresponding to the space–time dimension and ℓ denoting the length scale in this geometry. These geometries are characterized by the parameter z known as the dynamical or critical exponent. For $z=1$ this space–time is the usual anti-de Sitter (AdS) metric in Poincaré coordinates with the AdS radius ℓ. According to the AdS/CFT correspondence [2] and its generalizations beyond high energy physics, these metrics are important in the search for gravity duals of systems that appear in condensed matter physics and in quantum chromodynamics [1,3]. There are many theories of interest when studying such critical points, such theories exhibit the anisotropic scale invariance $t \to \lambda^z t$, $\vec{x} \to \lambda\vec{x}$, where the dynamical exponent z accounts for the different scale transformation between the temporal and spatial coordinates, and they are of particular interest in studies of critical exponent theory and phase transitions. The Lifshitz space–time exhibits these symmetries along with the scaling of the radial coordinate $r \to \lambda^{-1}r$. On the other hand, gravity duals of field theories with an anisotropic scale invariance at a finite temperature are represented by black hole geometries whose asymptotic behavior is given by (1.1), known as asymptotically Lifshitz black holes. Several Lifshitz black hole metrics have been found, which are solutions of Einstein gravity with several matter fields and also of gravity theories in vacuum with higher curvature corrections, such as $f(R)$, Lovelock, New Massive Gravity and others (see for instance [4–13]).

Asymptotically Lifshitz black holes can also emerge in Conformal Weyl Gravity in four dimensions with dynamical exponents

E-mail addresses: fherrera@userena.cl (F. Herrera), yvasquez@userena.cl (Y. Vásquez).

https://doi.org/10.1016/j.physletb.2018.05.051
0370-2693/

$z = 0$ and $z = 4$ [14]. Conformal Gravity (CG) in four dimensions is a higher derivative theory of gravity, whose action is given by the square of the Weyl tensor, and it is invariant under conformal transformations of the metric tensor:

$$g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x) , \tag{1.2}$$

where the conformal factor Ω is a function of the coordinates. A consequence of this is that CG is sensitive to angles but not to distances. CG is naturally subject to conformal anomaly, and renormalizability requires the inclusion of Ricci squared terms. On the other hand, Einstein gravity in four dimensions is two-loop divergent [17], and is perturbatively renormalizable when extended by the inclusion of curvature squared terms in the Lagrangian [15,16]; moreover, in contrast to Einstein gravity, CG contains ghost-like degrees of freedom in the form of massive spin-2 excitations, which implies vacuum instability. However, Einstein gravity with a cosmological constant is equivalent, at the tree level, to four-dimensional CG with Neumann boundary conditions [18]. The equivalence of the two theories, CG and Einstein gravity, was also studied in [19], both at the level of the action and at the variation of it. Furthermore, an important characteristic is that any space–time conformal to an Einstein manifold is a solution of CG and, because the equations of motion of CG contain fourth order derivatives, there are other solutions to CG than just Einstein gravity. Another connection between both theories is that the renormalized action of four-dimensional Einstein gravity in asymptotically hyperbolic Einstein spaces is on-shell equivalent to the action of CG [20]. The most general spherically symmetric asymptotically AdS solution of four-dimensional CG minimally coupled with an electromagnetic field is static and was obtained in [21]. Moreover, it has been argued that CG may be able to explain the galactic rotation curves without the assumption of dark matter [22]. Further black hole solutions to the field equations of four-dimensional CG in vacuum and in the presence of a Maxwell field include the general Kerr–NUT–AdS-like solution [23], charged and rotating AdS black holes [24], cylindrical black hole solutions [25], and also black hole solutions with both Abelian electromagnetic and $SU(2)$ Yang–Mills fields were obtained in [26]. Another consequence of the invariance of CG under the transformation (1.2) is that the gravitational field equations of CG, which in four space–time dimensions are given by the vanishing of the Bach tensor, imply that the trace of the energy-momentum tensor of the matter sources must vanish. This is because the Bach tensor is traceless; therefore, among the matter sources which can be coupled to CG, one must consider conformally invariant fields such as a minimally coupled electromagnetic field, SU(2) Yang–Mills fields, or a conformally invariant scalar field. It is worth stressing that only a few solutions to CG with a conformally coupled scalar field have been obtained in the literature. Among them, we can mention boson stars with a self-interacting complex scalar field [27] and a C-metric with a conformally coupled scalar field [28]. On the other hand, conformally invariant theories of gravity also exist in other space–time dimensions. In three space–time dimensions the equations of motion of CG contain third derivatives of the metric, and some black hole solutions have been studied in [29–31]. Additionally, three-dimensional Lifshitz black holes with $z = 0$ were obtained in [32]. In six space–time dimensions, AdS and Lifshitz black hole solutions of CG in vacuum and also coupled to conformal matter were obtained in [33].

According to the AdS/CFT correspondence, the search for black hole solutions to theories of gravity coupled with scalar fields and electromagnetic fields is an important task in the studies of holographic superconductors [34], which is dual to a system consisting of a black hole with a scalar field minimally coupled to gravity, which below a critical temperature condenses outside the black hole horizon. The dual description of a superconductor also works for scalar fields non-minimally coupled to gravity [35]. Further, Lifshitz holographic superconductivity has been the topic of numerous studies with interesting properties when the duality principle is generalized to non-relativistic systems [3,36]. In addition to the recent applications of the gauge/gravity duality to condense matter phenomena like superconductivity, the behavior of scalar fields outside black holes has been extensively studied over the years, mainly in connection with the no-hair theorems (for a review on this topic see [37–39]). The basic physical requirement of a black hole with scalar hair is the scalar field must be regular on the horizon and fall off sufficiently rapidly at infinity. From the above discussion, we conclude that it is important to understand the behavior of matter fields all the way from the black hole horizon to asymptotic infinity.

An important characteristic of black holes is their proper (quasinormal) oscillations. The quasinormal modes (QNMs) have a long history [40–45], and nowadays are of great interest due to the observation of gravitational waves from the merger of two black holes [46]. The QNMs give information about the stability of the black holes and also of probe matter fields that evolve perturbatively in the exterior region of black holes, without backreaction on the metric. Also, the QNMs are important in the context of the gravity/gauge duality, because they give information about the relaxation times of thermal states in the boundary theory [47], where the relaxation time is proportional to the inverse of the smallest imaginary part of the quasinormal frequencies (QNFs). On the other hand, in the context of black hole thermodynamics, the quantum area spectrum [48] as well as the mass and entropy spectrum can be determined through the knowledge of the QNMs [49–51]. The QNMs for asymptotically Lifshitz black holes have been studied for different types of perturbations. The QNMs for scalar field perturbations on the background of Lifshiftz black holes have been analyzed in [32,52–62], and generally the scalar modes are stable. Also, the propagation of fermionic perturbations and the QNMs of Lifshitz black holes were studied in Refs. [52,63–65]. The electromagnetic QNMs were investigated in [66]. Furthermore, the QNMs of Lifshitz black holes with hyperscaling violation have been considered in Refs. [67–69].

The aim of this paper is to find asymptotically AdS black hole solutions and asymptotically Lifshitz black hole solutions to a gravitational system, which for simplicity consists only of a real scalar field conformally coupled to CG in four dimensions, which is an important issue due to their potential applications in the description of dual field theories at a finite temperature through the gauge/gravity duality to condense matter phenomena. We require the scalar field to be regular on the horizon and to fall off at spatial infinity. We also assume spherical symmetry of the system. Then, the thermodynamical properties of the solutions are studied, such as their mass, entropy and temperature and it is verified if the first law holds. Furthermore, we study the propagation of a conformally coupled test scalar field in the black hole solutions found and determine the QNMs analytically in several cases. Remarkably, such as in the free source case, we find that there are analytical AdS black hole and Lifshitz black hole solutions with dynamical exponents $z = 0$ and $z = 4$. On the other hand, calculating conserved charges of Lifshitz black holes is a difficult task; however, progress was made on the computation of the mass and the related thermodynamics quantities by using the Abbott–Deser–Tekin (ADT) method [70,71] and its off-shell generalization [72,73], as well as the Euclidean action approach [54,74]. In order to calculate the conserved charges, we employ the generalized ADT formalism proposed in [75], which corresponds to the off-shell generalization of the on-shell Noether potential of the ADT formalism [76,77]. We