Physics Letters B 782 (2018) 346–352

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Kinks in higher derivative scalar field theory

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ARTICLE INFO

Article history:
Received 12 April 2018
Accepted 18 May 2018
Available online 21 May 2018
Editor: M. Cvetič

Keywords:
Higher-order derivative scalar field
Kinks
Linear stability

ABSTRACT

We study static kink configurations in a type of two-dimensional higher derivative scalar field theory whose Lagrangian contains second-order derivative terms of the field. The linear fluctuation around arbitrary static kink solutions is analyzed. We find that, the linear spectrum can be described by a supersymmetric quantum mechanics problem, and the criteria for stable static solutions can be given analytically. We also construct a superpotential formalism for finding analytical static kink solutions. Using this formalism we first reproduce some existed solutions and then offer a new solution. The properties of our solution is studied and compared with those preexisted. We also show the possibility in constructing twinlike model in the higher derivative theory, and give the consistency conditions for twinlike models corresponding to the canonical scalar field theory.

1. Introduction

Kink is the simplest topological defect. It exists in nonlinear scalar field theories with at least two degenerated vacua, and has been studied in many branches of physics [1]. In the early study of kinks, the scalar field theory is assumed to be canonical, and its Lagrangian can be written as $\mathcal{L}_0 = X - V(\phi)$. Here $X \equiv -\frac{1}{2}(\partial_\mu\phi)^2$ represents the standard kinetic term. In this simple theory, kink solutions can be obtained by choosing suitable scalar potentials. Two well-known solutions are the Z_2 symmetric ϕ^4 kink and the periodic sine-Gordon kink [1].

These two solutions have many differences. For example, the sine-Gordon model supports the interesting breather solution, while in the ϕ^4 model one can only find an approximation to the breather solution called oscillon, which emits radiation [2,3]. The oscillon solutions can also be found numerically in many other models with higher-order polynomial scalar potentials, such as ϕ^6 [4] and ϕ^8 potentials [5]. Another difference between the two models lies in their linear perturbation spectra. The ϕ^4 model has two bound states: a zero mode and a massive excitation, but the sine-Gordon model only has a zero mode. The massive excitation of the ϕ^4 model leads to the bounce windows when two kinks collide [2].

Recently, with the development of cosmology, many non-canonical scalar field theories were proposed [6–10]. In a typical noncanonical scalar field theory (dubbed as the K-field theory), the Lagrangian is assumed to be an arbitrary function of ϕ and X. This theory was originally applied in cosmology [11–13], and later was used to construct kink solutions either in two-dimensional Minkowski space [14–18], or in five-dimensional warped space [19–23]. In order to find analytical kink solutions in K-field theory, one can use the superpotential method, which rewrites the original second-order differential equations into some first-order ones by introducing the so-called superpotential (see for example Refs. [18–20,22]). The linear perturbation of static K-field kinks was systematically investigated in Refs. [18,21].

The Lagrangian of K-field contains only ϕ and its first-order derivative X. It is natural to ask can we extend the K-field Lagrangian by adding the second-order derivatives of ϕ, such as $Y \equiv \partial_\mu\partial^\mu\phi$? In fact, this is not a new idea. The study of higher-order derivative theories dates back to the nineteenth century [24], and the result is now concluded as the Ostrogradski's theorem, which states that all the Hamiltonians of non-degenerate higher time derivative theory suffer from linear instabilities (for more details see Refs. [25,26]). This instability can be avoided in some special models whose equations of motion are second order despite the presence of higher-order derivatives in the Lagrangians. A well-known example is the Galileon field [27], whose Lagrangian takes the following form in $1+1$ dimensions:

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi + \alpha\partial_\mu\phi\partial^\mu\phi\Box\phi. \tag{1}$$

Soliton solutions in Galileon field theory have been explored in Refs. [28–32]. Especially, by using a zero-mode argument, the au-

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https://doi.org/10.1016/j.physletb.2018.05.048

thors of Ref. [28] showed that the Galileon field cannot give rise to static solitonic solutions.

Thus, in order to find static kink solutions in higher derivative theory, one needs to extend the Galileon theory. In four-dimensional curved space–time, the most general scalar-tensor theory with second-order equations of motion is the Horndeski theory [33]. But later it was realized that second-order equation is not mandatory for avoiding the Ostrogradski's instability. The Ostrogradski's instability can also be eliminated by introducing constraints [34,35], or in multifield models [36]. Nowadays, the most general extensions to the Horndeski's theory are the so-called degenerate higher-order scalar-tensor (DHOST) theories [37–39].

It is interesting to study the static kink solutions in various kinds of higher derivative scalar field theories, and see how the higher derivative terms affect the well-known properties of the canonical kinks. Some successful examples can be found in [40, 41]. Both works considered the so-called generalized Galileon theory [42], and the corresponding Lagrangian in two-dimensional Minkowski space reads

$$\mathcal{L} = f_1(\phi, X) + f_2(\phi, X)Y. \tag{2}$$

In this paper, we extend the works of Refs. [40,41] to a model with the following Lagrangian

$$\mathcal{L} = \mathcal{L}(\phi, X, Y). \tag{3}$$

This Lagrangian can be regarded as a simple subclass of the DHOST theories, and the corresponding equation of motion reads

$$\mathcal{L}_\phi + \partial^\mu(\mathcal{L}_X\partial_\mu\phi) + \partial^\mu\partial_\mu\mathcal{L}_Y = 0. \tag{4}$$

Here we have defined $\mathcal{L}_\phi \equiv \frac{\partial\mathcal{L}}{\partial\phi}$, and so on. Our aim is to find static kink solutions in a two-dimensional Minkowski space–time with line element $ds^2 = -dt^2 + dx^2$.

The paper is organized as follows. In the next section, we firstly give a general discussion on the linear stability of an arbitrary static solution of Eq. (4). We will show that under some conditions, the perturbation equation can be written as a factorizable Schrödinger-like equation, which ensures the stability of the solution. In Sec. 3, we construct the superpotential formalism corresponding to our model. This formalism is powerful in finding kink solutions. As examples, we will apply it to reproduce some of the solutions of [40], and then give our own solution. After that, we will consider, in Sec. 4, a constrained system. The constraint forces the equation of the higher derivative theory taking the same form as the one of the canonical theory. In this case, nonlinear terms of Y are allowed if some conditions were satisfied. We will also derive the equations that $\mathcal{L}(\phi, X, Y)$ has to satisfy in order to be a twinlike model of $\mathcal{L}_0$. Our results will be summarized in Sec. 5.

2. Linear stability of static configuration

Suppose we have obtained a static solution $\phi_c(x)$ of Eq. (4), it is important to consider the behavior of a small perturbation $\delta\phi(t,x) = \sum_{n=0}^{\infty}\psi_n(x)e^{i\omega_n t}$ around $\phi_c(x)$. The spectrum of ω_n can be obtained by solving the linear perturbation equation. Obviously, a real ω_n corresponds to a stable oscillation $\delta\phi(t,x)$, with frequency ω, around $\phi_c(x)$. While, an imaginary ω_n corresponds to an exponentially growing perturbation, and would destroy the original configuration $\phi_c(x)$. Therefore, when $\omega_n^2 \geq 0$ holds for all n, we say that the static configuration $\phi_c(x)$ is stable against small perturbation. Otherwise, $\phi_c(x)$ is unstable.

In Ref. [40], Bazeia et al. analyzed the linear perturbation of a model described by the Lagrangian (2). In this section, we will consider the linearization of static solution of model (3). To derive the linear equation of $\delta\phi(t,x)$, one can expand the action around $\phi_c(x)$ up to the second order of the perturbation:

$$\begin{aligned}\delta^{(2)}\mathcal{L} =& \mathcal{L}_X\delta^{(2)}X + \frac{1}{2}\mathcal{L}_{\phi\phi}(\delta\phi)^2 + \frac{1}{2}\mathcal{L}_{XX}(\delta^{(1)}X)^2\\ &+ \frac{1}{2}\mathcal{L}_{YY}(\delta Y)^2 + \mathcal{L}_{\phi X}\delta\phi\delta^{(1)}X + \mathcal{L}_{XY}\delta^{(1)}X\delta Y\\ &+ \mathcal{L}_{\phi Y}\delta\phi\delta Y + \mathcal{O}(\delta\phi^3).\end{aligned} \tag{5}$$

Here we have defined the following quantities:

$$\delta^{(1)}X = -(\partial^\mu\delta\phi)(\partial_\mu\phi) = -\delta\phi'\phi', \tag{6}$$

$$\delta^{(2)}X = -\frac{1}{2}(\partial^\mu\delta\phi)(\partial_\mu\delta\phi), \tag{7}$$

$$\delta Y = \partial^\mu\partial_\mu\delta\phi. \tag{8}$$

Obviously, the term $\frac{1}{2}\mathcal{L}_{YY}(\delta Y)^2$ inevitably leads to fourth-order derivatives terms in the linear perturbation equation. For simplicity, in this work we only consider the case with $\mathcal{L}_{YY} = 0$, so that the linear perturbation equation is second order. But it does not mean that $\mathcal{L}$ can only contain a linear term of Y. As we will see in Sec. 4, sometimes, $\mathcal{L}_{YY}$ is vanished after the background equation of motion is considered. In such case, nontrivial higher-order terms of Y are allowed, and do not change the final statements of this section.

At a first glance, the term $\mathcal{L}_{XY}\delta^{(1)}X\delta Y = -\mathcal{L}_{XY}(\partial^\mu\delta\phi)\times(\partial_\mu\phi)\partial^\nu\partial_\nu\delta\phi$ would also lead to a third-order derivative of $\delta\phi$ after an integration by parts. However, the higher-order derivative terms can be eliminated in the following sense:

$$\begin{aligned}\mathcal{L}_{XY}\delta^{(1)}X\delta Y =& -\frac{1}{2}\mathcal{L}_{XY}(\partial^\mu\delta\phi)(\partial_\mu\phi)\Box\delta\phi\\ &- \frac{1}{2}\mathcal{L}_{XY}(\partial^\mu\delta\phi)(\partial_\mu\phi)\Box\delta\phi\\ =& \frac{1}{2}\delta\phi\partial^\mu(\mathcal{L}_{XY}\partial_\mu\phi\Box\delta\phi) - \frac{1}{2}\delta\phi\Box(\mathcal{L}_{XY}\partial_\mu\phi\partial^\mu\delta\phi)\\ &+ \partial_\mu(\cdots),\end{aligned} \tag{9}$$

where the last term in the second line is a total derivative term. Obviously, the terms that contain $\partial^\mu\partial^\nu\partial_\nu\delta\phi$ are canceled.

In the end, for a static background kink configuration, the quadratic Lagrangian density of $\delta\phi$ reads

$$\begin{aligned}\delta^{(2)}\mathcal{L} =& \frac{1}{2}(\mathcal{L}_X + \mathcal{L}'_{XY}\phi' + \mathcal{L}_{XY}\phi'' + 2\mathcal{L}_{\phi Y})\delta\phi\Box\delta\phi\\ &+ \frac{1}{2}\mathcal{L}_{\phi\phi}(\delta\phi)^2 + \delta\phi\delta\phi'(\frac{1}{2}\mathcal{L}'_X - \frac{1}{2}\mathcal{L}'_{XX}\phi'^2 - \mathcal{L}_{XX}\phi'\phi''\\ &- \mathcal{L}_{\phi X}\phi' - \frac{1}{2}\mathcal{L}''_{XY}\phi' - \mathcal{L}'_{XY}\phi'' - \frac{1}{2}\mathcal{L}_{XY}\phi''')\\ &- \delta\phi\delta\phi''(\frac{1}{2}\mathcal{L}_{XX}\phi'^2 + \mathcal{L}'_{XY}\phi' + \mathcal{L}_{XY}\phi'').\end{aligned} \tag{10}$$

By defining the following variables

$$\mathcal{G} = \delta\phi\sqrt{\xi}, \tag{11}$$

$$z = \phi'\sqrt{\xi}, \tag{12}$$

$$\xi \equiv \mathcal{L}_X + \mathcal{L}'_{XY}\phi' + \mathcal{L}_{XY}\phi'' + 2\mathcal{L}_{\phi Y}, \tag{13}$$

$$\gamma = 1 - \frac{\phi'^2}{z^2}\left(2\mathcal{L}'_{XY}\phi' + \mathcal{L}_{XX}\phi'^2 + 2\mathcal{L}_{XY}\phi''\right), \tag{14}$$

the quadratic Lagrangian density can be simplified as

$$\delta^{(2)}\mathcal{L} = \frac{1}{2}\left\{-\mathcal{G}\partial_t^2\mathcal{G} + \mathcal{V}(x)\mathcal{G}^2 + \gamma\mathcal{G}\mathcal{G}''\right\}, \tag{15}$$