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Multichannel conformal blocks for scattering amplitudes

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ABSTRACT

By performing resummation of small fermion–antifermion pairs within the pentagon form factor program to scattering amplitudes in planar $\mathcal{N}=4$ superYang–Mills theory, we construct multichannel conformal blocks within the flux-tube picture for N-sided NMHV polygons. This procedure is equivalent to summation of descendants of conformal primaries in the OPE framework. The resulting conformal partial waves are determined by multivariable hypergeometric series of Lauricella–Saran type.

1. Introduction

Symmetries of a system allow one to significantly reduce the number of degrees of freedom that require dynamical considerations. Conformal block decomposition of correlation functions $\langle \prod_j \mathcal{O}_j \rangle$ of local operators $\mathcal{O}_j \equiv \mathcal{O}_j(z_j)$ is a way of implementing them in a scale-invariant field theory (or CFT) via the operator product expansion (OPE). Under the assumption of convergence, a correlator can be expanded in a complete set of primary operators Φ_{Δ_ℓ} of increasing scaling dimension and spin (cumulatively called Δ_ℓ) and their conformal descendants built with the action of derivatives $\partial^n \Phi_\ell$. It is the latter infinite tower which is conveniently packed together in the conformal block, also known as the partial wave $\mathcal{F}_{\boldsymbol{\Delta}}(\boldsymbol{w})$, which is a function of $\boldsymbol{\Delta} = \{\Delta_\ell\}$ and cross ratios $\boldsymbol{w} = \{w_\ell\}$, schematically,

$$\langle \prod_j \mathcal{O}_j \rangle = \left(\prod_{j<k} z_{jk}^{\Delta_{jk}} \right) \sum_{\boldsymbol{\Delta}} a_{\boldsymbol{\Delta}} \mathcal{F}_{\boldsymbol{\Delta}}(\boldsymbol{w}) \,, \tag{1}$$

with an overall multiplicative function of the coordinate differences with powers Δ_{jk} being functions of the operator $\mathcal{O}_j$ dimensions/spins conveniently chosen to carry the scaling dimension of the left-hand side. The conformal blocks $\mathcal{F}_{\boldsymbol{\Delta}}$ are eigenfunctions of conformal Casimir operators for successive channels in the operator product expansion and are subject to appropriate boundary conditions. While the low-point correlators are well studied, there is little to no knowledge of multichannel conformal blocks.

Conformal blocks are ubiquitous in physics so they make their natural appearance in the analysis of scattering amplitudes within the pentagon operator product expansion [1,2]. In the latter, one relies on a dual description of amplitudes in terms of excitations propagating on a color flux-tube sourced by the contour of the Wilson loop living in the four-dimensional momentum space [3–8]. The vacuum represented by the flux is in fact SL(2) invariant to lowest order in 't Hooft coupling [9,10]. This property was used in the construction of conformal blocks for (N)MHV hexagons and heptagons [11–14].

The tree-level N-particle ratio function of the NMHV to MHV tree amplitudes

$$\mathbb{R}_N = \sum_{1<j<k<N-2} [1, j, j+1, k, k+2] \tag{2}$$

is determined by the R-invariants [15,16]

$$[i, j, k, l, m] = \frac{\delta^{0|4}(\chi_i(jklm) + \text{cyclic})}{(ijkl)(jklm)(klmi)(lmij)(mijk)} \,, \tag{3}$$

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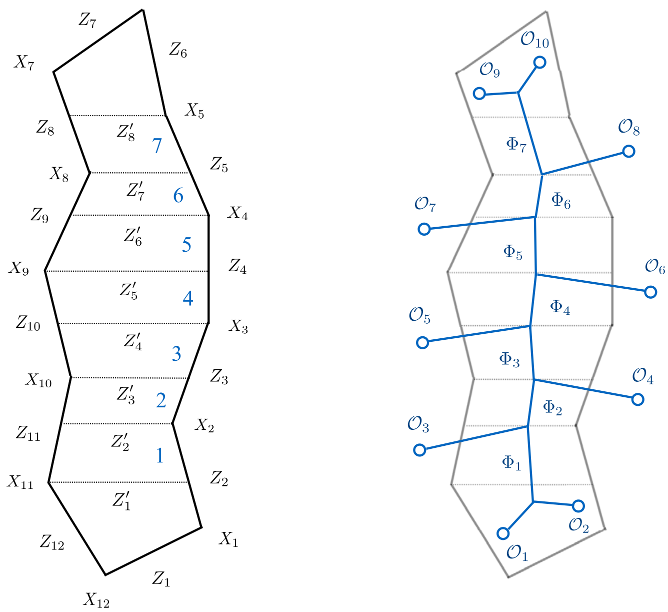

Fig. 1. A tessellation of a polygon (dodecagon on the left) and its OPE dual graph (on the right).

with the four-bracket defined by the determinant $(jklm) \equiv \varepsilon_{JKLM} Z_j^J Z_k^K Z_l^L Z_m^M$ built from the momentum twistors Z_j^J and χ_j^A being their fermionic partners. Within the pentagon form factor program, each individual Grassmann component $R^{[r_1,r_2,\dots,r_{N-5}]}$ of $\mathbb{R}_N$, with R-weights $r_1,\dots,r_{N-5}$ of all parent excitations, corresponding to the SU(4) dimensions $\mathbf{R}$ of flux-tube excitations, $r=0,1,2,3,4$ for $\mathbf{R}=\bar{\mathbf{1}},\bar{\mathbf{4}},\mathbf{6},\mathbf{4},\mathbf{1}$, can be represented in terms of flux-tube integrals

$$
\begin{aligned}
R^{[r_1,r_2,\dots,r_{N-5}]} = & \sum_{\alpha_1,\dots,\alpha_{N-5}} e^{-t_{\alpha_1}\tau_1 - \dots - t_{N-5}\tau_{\alpha_{N-5}} + ih_{\alpha_1}\varphi_1 + \dots + ih_{\alpha_{N-5}}\varphi_{N-5}} \\
& \times \int \prod_{j=1}^{N-5} \frac{du_j}{2\pi} e^{2i\sigma_1 u_1 + \dots 2i\sigma_{N-5} u_{N-5}} I^{\mathbf{R}_1|\dots|\mathbf{R}_{N-5}}(\alpha_1, u_1|\dots|\alpha_{N-5}, u_{N-5}),
\end{aligned}
\tag{4}
$$

where the $3(N-5)$ conformal invariants of Eq. (2) were traded for $N-5$ sets of triplets $(\tau_j, \sigma_j, \varphi_j)$ with their reciprocal variables interpreted as the energy (or twist), momentum and helicity, respectively, of the particles propagating on the flux and their SU(4) representation $\mathbf{R}_j$.

There is an infinite number of (parent) flux-tube excitations $\Phi_\alpha^{\mathbf{R}}$ [12,14] of different spin/R-change and increasing energy (i.e., conformal primary states, in the language of CFT) which determine the integrand $I^{\mathbf{R}_1|\dots|\mathbf{R}_{N-5}}$. Their descendants arise by gluing small fermion–antifermion pairs to $\Phi_\alpha^{\mathbf{R}}$'s. A small fermion–antifermion pair $\psi_s\bar{\psi}_s$ is equivalent to the derivative since ψ_s at zero momentum becomes the generator of Poincaré supersymmetry Q and since $\{Q,\bar{Q}\} \sim P$, according to their algebra, $(\psi_s\bar{\psi}_s)^n \Phi_\alpha^{\mathbf{R}} \sim \partial^n \Phi_\alpha^{\mathbf{R}}$ by analogy with conformal OPE alluded to above. In this note, we will construct multichannel conformal blocks for N-leg NMHV amplitudes by explicit resummation of the entire tower of small fermion–antifermions pairs accompanying parent particles, this will yield the substitution in the integrand

$$
\begin{aligned}
& I^{\mathbf{R}_1|\dots|\mathbf{R}_{N-5}}(\alpha_1, u_1|\dots|\alpha_{N-5}, u_{N-5}) \\
& \to I^{\mathbf{R}_1|\dots|\mathbf{R}_{N-5}}(\alpha_1, u_1|\dots|\alpha_{N-5}, u_{N-5}) \mathcal{F}^{[r_1,\dots,r_{N-5}]}_{h_{\alpha_1},t_{\alpha_1}|\dots|h_{\alpha_{N-5}},t_{\alpha_{N-5}}}(u_1,\tau_1|\dots|u_{N-5},\tau_{N-5})
\end{aligned}
\tag{5}
$$

where $\mathcal{F}$ are the conformal blocks in question. This formalism is equivalent to the projection technique for computation of conventional conformal blocks in a CFT, which we briefly review by applying it to a four-point correlator in Appendix A to draw a parallel with the flux-tube physics.

2. Kinematics

Before turning to dynamics, let us introduce some kinematics first. The starting point is a tessellation of a polygon determined by the reference momentum twistors Z_j in terms of a sequence of squares formed by the polygon edges and internal light-like lines encoded in the momentum twistors Z'_k (see the left panel in Fig. 1 for the case of the dodecagon). A choice of a square automatically defines a conformal frame and thus a channel for propagation of parent flux excitations and their descendants. This is equivalent to a choice of an OPE channel for correlation functions (see the right panel in Fig. 1). To make the discussion more explicit, let us provide a choice of reference twistors for the dodecagon as a case of study (shown in Fig. 1)

$$
\begin{aligned}
& Z_1 = (6,4,12,5), \quad && Z_2 = (1,2,4,1), \quad && Z_3 = (0,1,1,0), \quad && Z_4 = (0,1,0,0), \\
& Z_5 = (0,2,-1,1), \quad && Z_6 = (-1,6,-4,6), \quad && Z_7 = (-4,6,-5,12), \quad && Z_8 = (-2,1,-1,4), \\
& Z_9 = (-1,0,0,1), \quad && Z_{10} = (1,0,0,0), \quad && Z_{11} = (2,0,1,1), \quad && Z_{12} = (6,1,6,4),
\end{aligned}
\tag{6}
$$