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Parity at the Planck scale

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ABSTRACT

We explore the possibility that well known properties of the parity operator, such as its idempotency and unitarity, might break down at the Planck scale. Parity might then do more than just swap right and left polarized states and reverse the sign of spatial momentum **k**: it might generate superpositions of right and left handed states, as well as mix momenta of different magnitudes. We lay down the general formalism, but also consider the concrete case of the Planck scale kinematics governed by κ-Poincaré symmetries, where some of the general features highlighted appear explicitly. We explore some of the observational implications for cosmological fluctuations. Different power spectra for right handed and left handed tensor modes might actually be a manifestation of deformed parity symmetry at the Planck scale. Moreover, scale-invariance and parity symmetry appear deeply interconnected.

1. Introduction

The search for the theory of quantum gravity has left us with the distinct possibility that familiar concepts, such as space–time manifolds, might completely dismantle at the Planck scale and be replaced by radically new structures (examples include non-commutative geometry, spin networks and multifractal theories [1–7]). This prospect should serve as a warning against importing intuition derived from low-energy concepts into the UV/short-scale description of space–time. A case in point is the concept of parity at the Planck scale, and the fate of some of its familiar and "self-evident" properties associated with conventional, low-energy space–time.

In elementary treatments (which ignore curvature, but can be extended to curved manifolds, for example using the tetrad formalism), the parity operator $\mathbb{P}$ is frequently introduced as a transformation driven by a "mirror" action applied to the spatial reference frame, sending $\mathbf{x}$ to $-\mathbf{x}$, and then observing the transformation laws of all quantities, classical or quantum, that live in that space. This simple characterization encounters obstructions in most quantum gravity treatments (even before curvature is introduced), where the position space picture is heavily eroded, with the arena of physics often shifting in the first instance to a non-trivial momentum space (in some cases *always* curved, regardless of the effects of gravity), from which it is difficult, if not outright impossible, to derive a position space counterpart.

Similar concerns arise when analysing parity on more general grounds, as a transformation belonging to the Lorentz group that can be defined without making explicit reference to position space, via its action on the generators of the Poincaré algebra. Also in this picture, the fact that in the quantum gravity regime the Lorentz and/or the translation sector of the Poincaré algebra might be deformed (for example because of the non-trivial geometry of momentum space mentioned above) motivates us to expect that parity transformations acquire non-trivial properties.[1] Should the mathematical treatment point us in that direction, we should therefore not be afraid to eschew properties which are self-evident within the prejudiced intuitions associated with position space and its standard (local) symmetries.

In this paper we take aim at two such dogmas. Firstly, we challenge the idea that $\mathbb{P}^2 = 1$ is a logical necessity. That parity must

[1] Even more reasons of concern arise in looking at parity in a curved spacetime setting. In this case in fact one would normally define parity via the transformation properties of tetrads rather than of spatial coordinates or of local symmetry generators. However the nontrivial nature of the momentum space is an obstacle to the very definition of tetrads, that is still a matter of active research [8,9]. Since in this work we are concerned with quantum deformations of "flat" spacetime (in the absence of gravity), we leave further comments on this matter to the concluding outlook.

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be idempotent is obvious from the fact that two mirror transformations upon $\mathbf{x}$ take us back to the original frame. But once we abandon parity as a concept driven by a conventional pre-fixed smooth space, this need not be the case. In fact we find that lost idempotency of $\mathbb{P}$ is a feature of models with κ-Poincaré relativistic symmetries.

Secondly, we question whether parity merely swaps right-handed and left-handed particle states, whilst reversing their momentum. This property usually results from combining the idempotency and the unitarity (and thus hermiticity) of parity. Both could break down at the Planck scale. In either case parity would then map a right-handed state into a quantum superposition of right- and left-handed states. It might also bring the momentum $\mathbf{k}$ non-trivially into the operation, and map a $\mathbf{k}$ into a momentum of different modulus.

In either case, the observational implications could be dramatic, should we have direct access to Planck scale physics, at least via thought experiments. More mundanely, we could see parity at the Planck scale transmuted into parity violating tensor fluctuations left over from the early universe, as we show in this paper.

The plan of this paper is as follows. In Section 2 we lay down the general framework for what might be the effects of parity at the Planck scale, and the most direct observational implications. For the rest of the paper we then illustrate *some* of the properties encoded into this general framework with reference to concrete theories of quantum gravity, or associated. Specifically, in Section 3, we briefly introduce the κ-Poincaré algebra, a deformation of the ordinary algebra of relativistic symmetries that models putative quantum gravity effects at the Planck scale. We show how parity is non-trivially affected by the deformation of the algebra leading, as mentioned above, to an action of the parity operator on translation generators that no longer squares to one. We then explore the effects of the deformation on the helicity operator and show how parity swaps right handed and left handed states changing their spatial momentum. This leads to an interesting connection between parity invariance and scale invariance which we explore at the end of Section 3. A discussion of the results is presented in the concluding Section 4.

2. General framework and its phenomenology

We start by setting up the general framework for what could be the effects of parity at the Planck scale, should some of its basic properties be lost. We will at first do this without reference to any specific theory (even though we mention possible sources for the effects), keeping the discussion as general as possible.

It is usually the case that $\mathbb{P}^2=1$, so that hermiticity and unitarity are equivalent: $\mathbb{P}^\dagger=\mathbb{P}=\mathbb{P}^{-1}$. Given that $\mathbb{P}^2=1$, its action on 2-dimensional vectors (such as the eigenvectors of helicity or of parity itself) can be encoded by a generic square root of the unit matrix (there are 4 such solutions):

$$\alpha_{ij}=\begin{bmatrix} a & \frac{1-a^2}{b} \\ b & -a \end{bmatrix}, \tag{1}$$

where a and b are any complex numbers, to begin with. Then, hermiticity forces a to be real and $a^2+|b|^2=1$, so that:

$$\alpha_{ij}=\begin{bmatrix} a & b^\star \\ b & -a \end{bmatrix} \quad , \quad a^2+|b|^2=1. \tag{2}$$

If we are in the eigenbasis of parity itself then $b=0$ and a can be set to 1. If we are in the eigenbasis of the helicity, then $a=0$ and b can be at most a phase. Usually one sets $b=1$, so that

$$\alpha_{ij}=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{3}$$

The action of parity on a state with right/left (R/L) helicity therefore can only result in a state with opposite helicity, that is, parity merely swaps states with R and L helicities.

The situation is entirely different should the hermiticity and/or unitarity of the parity operator be lost (for example because they are defined with respect to a non-trivial measure [10], so that with regards to the usual one they appear violated). This could happen:

- with $\mathbb{P}^2=1$ preserved, so that if we break one of hermiticity and unitarity we must break both (we *may* still preserve *both*, of course).
- or with $\mathbb{P}^2\neq 1$, in which case we are forced to break one of hermiticity or unitarity (or both, if we so wish).

In either case parity can then map a R-helicity state into a superposition of R and L states, and likewise for L:

$$\mathbb{P}|R\rangle=\alpha_{RR}|R\rangle+\alpha_{RL}|L\rangle \tag{4}$$

$$\mathbb{P}|L\rangle=\alpha_{LR}|R\rangle+\alpha_{LL}|L\rangle\,. \tag{5}$$

If $\mathbb{P}^2\neq 1$ and parity is still unitary (but not hermitian) this is because there is no constraint on the rotation angle of the unitary operation. In this case parity invariance may become an infinite set of conditions, unless $\mathbb{P}^n=1$ for some integer (in which case there are $n-1$ conditions). If $\mathbb{P}^2=1$, but its hermiticity and unitary are lost, then parity is no longer an observable, so there is no point in seeking its eigenbasis. However helicity may still be an observable, in which case the action of parity upon the induced orthonormal basis is the most general matrix envisaged in Eq. (4).

More generally, we should allow for the possibility that parity does not factor the internal space and the momentum $\mathbf{k}$. Usually parity sends $\mathbf{k}$ to $-\mathbf{k}$ regardless of what it does to R and L states, and this could continue to the case at the Planck scale, even if (4) is non-trivial. But the action on momentum space could also be more complicated, and not factor it out of the action upon R and L. In general we should consider the matrix:

$$\mathbb{P}|i,\mathbf{k}\rangle=\sum_{j,\mathbf{k}'}\alpha_{ij}(\mathbf{k},\mathbf{k}')|j,\mathbf{k}'\rangle, \tag{6}$$

with $i,j=R/L$. The usual action of parity corresponds to:

$$\alpha_{ij}(\mathbf{k},\mathbf{k}')=\delta(\mathbf{k}+\mathbf{k}')\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{7}$$

but this need not be true at the Planck scale.

The different action of parity upon states would have immediate phenomenological implications, should the fluctuations of our Universe have their origin in quantum vacuum fluctuations. The vacuum expectation value of a given field may be seen as the norm of its one particle states (see [11] and references therein):

$$\langle\mathbf{k},R|\mathbf{k}',R\rangle=\delta(\mathbf{k}-\mathbf{k}')\mathcal{P}_R(k) \tag{8}$$

and likewise for L. The way parity invariance usually forces $\mathcal{P}_R(k)=\mathcal{P}_L(k)$ is by the following chain:

$$\langle\mathbf{k},R|\mathbf{k}',R\rangle=\mathbb{P}\left(\langle\mathbf{k},R|\mathbf{k}',R\rangle\right)=\langle-\mathbf{k},L|-\mathbf{k}'L\rangle \tag{9}$$

after which isotropy leads to $\mathcal{P}_R(k)=\mathcal{P}_L(k)$ (since $|\mathbf{k}|=|-\mathbf{k}|$). This argument breaks down should parity act non-trivially, as we shall now see. We separate two extreme cases, one involving purely the internal space, the other purely momentum space. In general these two cases could appear combined and even interact non-trivially.