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The cosmological model with a wormhole and Hawking temperature near apparent horizon

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ABSTRACT

In this paper, a cosmological model with an isotropic form of the Morris–Thorne type wormhole was derived in a similar way to the McVittie solution to the black hole in the expanding universe. By solving Einstein's field equation with plausible matter distribution, we found the exact solution of the wormhole embedded in Friedmann–Lemaître–Robertson–Walker universe. We also found the apparent cosmological horizons from the redefined metric and analyzed the geometric natures, including causal and dynamic structures. The Hawking temperature for thermal radiation was obtained by the WKB approximation using the Hamilton–Jacobi equation and Hamilton's equation, near the apparent cosmological horizon.

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1. Introduction

The solution to the black hole embedded in expanding universe has been familiar to relativists and cosmologists for a long time since McVittie derived a model [1]. The cosmological black hole solutions are more like a realistic model. Recently, the solutions also has attracted us because of the role of black hole in the expanding cosmological model. The evidences for the accelerating universe and dark energy forced the study of the cosmological black hole model. The interaction of black holes with dark energy distributed over the universe can be one of the most important issues. Moreover, they can show a generalized theory of global and local physics, that is interested in the unification of interactions [2].

After McVittie solution there were several models of black hole in the universe. McVittie spacetime [1] was a spherically symmetric, shear free, perfect solution and was asymptotically Friedmann–Lemaître–Robertson–Walker (FLRW) model. In this model, matters were isotropically distributed and there was no-accretion onto the black hole centered at the FLRW universe. As generalizations of the McVittie solution, there were solutions of the charged black hole in expanding universe [3,4]. Faraoni and Jacques obtained a generalized McVittie solution without no-accretion condition [2]. Sultana–Dyer [5] got the extension of the geometry generated by conformally transforming the Schwarzschild metric with the scale factor of flat FLRW universe. Kottler [6] derived the solution of the Schwarzschild black hole in de Sitter background.

The research on wormhole is also an important issue in study of spacetime physics. The wormhole usually consists of exotic matter which satisfies the flare-out condition and violates weak energy condition [7,8], even though there have been attempts to construct wormhole with non-exotic matter [9]. There were also solutions of cosmological wormhole model as well as the cosmological black hole solutions. There was the solution of a wormhole in inflationary expanding universe model [10]. In this solution, the wormhole throat inflates at the same rate as that of the scale factor. Also there was a wormhole solution in FLRW cosmological model [11]. The solution also showed the expansion of the wormhole throat at the same rate as that of the scale factor. Hochberg and Kepart tried to extend the Visser type wormhole into a surgical connection of two FLRW cosmological models [12]. Similarly there was a solution for the connection of two copies of Schwarzschild–de Sitter type wormhole as the cosmological wormhole model [13]. There was a research on quantum cosmological approach by considering wave function of the de Sitter cosmological model with a wormhole [14]. Recently there was a cosmological wormhole solution [15] as a generalization of MT wormhole in FLRW universe, but there was a weak point that Einstein's equation could not be guaranteed.

First of all, it is necessary to find the exact solution of the cosmological wormhole model satisfying Einstein field equation. For reasons similar to black holes, the exact wormhole model embedded in the universe is very interesting to us. They will provide a lot of information to understand the relationship between wormhole matter and the background spacetime. Most of the previous cosmological wormhole solutions originated from the spacetime assumed to be plausible models. If a wormhole were in the ex-

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panding universe, it would interact with dark energy in some way that would be considered. The impacts on the spacetime are also very interesting. So we need to find the exact solution of wormhole universe to see its influence to the evolution of spacetime.

In the spacetime structure with strong gravity, the Hawking temperature is one of the issues focused on gravitation problem dealing with quantum phenomena. The Hawking temperature, derived from the definition of surface gravity at event horizon, is a good example of the semi-classical handling of quantum gravity. Usually the temperature is calculated from the surface gravity defined by Killing vector in static case. In the dynamic case, such as a black hole or a wormhole in expanding universe, the surface gravity at the event horizon is not constant. In this case, we need to adopt the Kodama vector [16] instead of the Killing vector, and try to find the corresponding physical quantities for the spacetime [17]. By using the WKB approximation to the tunneling method, the Hawking temperature is derived by comparing the thermal distribution and probability amplitude from Hamilton–Jacobi equation [18]. There is also another way to find the probability amplitude from Hamilton's equation, which was designed by Parikh and Wilczek [19]. There is an example of Hawking temperature at apparent horizon of the FLRW model in both methods [20].

In this paper, we have found the exact solution of the wormhole embedded in FLRW universe, the locations and the existence conditions of apparent horizon. The influence of wormhole matter to the structure of the apparent horizons was studied. The Hawking radiation was also discussed, and the temperature was also derived near the apparent horizon.

2. Cosmological wormhole model

2.1. Isotropic wormhole

Now we derive the exact model of the wormhole embedded in FLRW cosmology. First, we need to find the isotropic form of the wormhole model in order to derive the wormhole solution embedded in a cosmological model because of the isotropy of the cosmological models in this paper. The Morris–Thorne type wormhole (MT-wormhole) is given by [7]

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{1-b(r)/r}dr^2 + r^2 d\Omega^2, \tag{1}$$

where $\Phi(r)$ is red-shift function and $b(r)$ is the shape function. The geometric unit, that is, $G = c = \hbar = 1$ is used here. The radial coordinate r is in the range of $b < r < \infty$. Two functions $\Phi(r)$ and $b(r)$ are restricted by the 'flare-out condition' to maintain the shape of the wormhole. Because the wormhole has the structure that prevents the existence of the event horizon, wormhole can be used for two-way travel. Since MT-wormhole is spherically symmetric form, we introduce the new coordinates $(\tilde{t}, \tilde{r})$ to define the isotropic form of a wormhole as

$$ds^2 = -A^2 d\tilde{t}^2 + B^2(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2). \tag{2}$$

In this paper, we treat the example of $b(r) = \frac{b_0^2}{r}$ and $e^{2\Phi} = 1$ to see the nature of wormhole geometry more simply. Then B becomes

$$B = \frac{2}{1+\sqrt{1-\frac{b_0^2}{r^2}}}, \tag{3}$$

where the integration constant is determined by the asymptotically flat condition. The new coordinate $\tilde{r}$ is given in terms of old coordinates r as

$$\tilde{r} = \frac{r}{B} = \frac{1}{2}(r + \sqrt{r^2 - b_0^2}), \quad (b_0/2 < \tilde{r} < \infty). \tag{4}$$

The old coordinate r and the radial factor B are given in terms of $\tilde{r}$ as

$$r = \tilde{r} + \frac{b_0^2}{4\tilde{r}}, \qquad B = \frac{r}{\tilde{r}} = \left(1 + \frac{b_0^2}{4\tilde{r}^2}\right). \tag{5}$$

Thus the isotropic form of the static wormhole is

$$ds^2 = -d\tilde{t}^2 + \left(1 + \frac{b_0^2}{4\tilde{r}^2}\right)^2 (d\tilde{r}^2 + \tilde{r}^2 d\Omega^2), \tag{6}$$

for the ultra-static case. In this isotropic wormhole model, the metric does not diverge at $\tilde{r} = b_0/2$, while it diverges at $r = b_0$ in MT-wormhole. This is because this non-diversity is removed by the coordinate transformation. We can also transform the matter part of the old model [21] into new one by using the relationship between two coordinates (5). For wormhole solution, the matter components in new coordinates are

$$T^{\tilde{0}}{}_{\tilde{0}} = -\frac{4^4 b_0^2 \tilde{r}^4}{8\pi(b_0^2 + 4\tilde{r}^2)^4} = \rho_w, \tag{7}$$

$$T^{\tilde{1}}{}_{\tilde{1}} = -\frac{4^4 b_0^2 \tilde{r}^4}{8\pi(b_0^2 + 4\tilde{r}^2)^4} = -\tau_w, \tag{8}$$

$$T^{\tilde{2}}{}_{\tilde{2}} = +\frac{4^4 b_0^2 \tilde{r}^4}{8\pi(b_0^2 + 4\tilde{r}^2)^4} = P_w, \tag{9}$$

$$T^{\tilde{3}}{}_{\tilde{3}} = +\frac{4^4 b_0^2 \tilde{r}^4}{8\pi(b_0^2 + 4\tilde{r}^2)^4} = P_w, \tag{10}$$

where ρ_w, τ_w, and P_w are wormhole energy density, tension, and pressure, respectively. The negative density is still required for the isotropic form of wormhole model.

2.2. Exact solution

From now on, we use un-tilde coordinate in isotropic form for convenience if only there are no confusions in the rest of this paper. The FLRW spacetime in isotropic form is given by [1]

$$ds^2 = -dt^2 + \frac{a^2(t)}{(1+kr^2)^2}(dr^2 + r^2 d\Omega^2). \tag{11}$$

Here $a(t)$ is the scale factor and $k = 1/4\mathcal{R}^2$, where $\mathcal{R}$ is the curvature, and k goes zero in case of flat FLRW spacetime. We start from the general isotropic metric element to see the unified wormhole in FLRW cosmological model as

$$ds^2 = -e^{\zeta(r,t)}dt^2 + e^{\nu(r,t)}(dr^2 + r^2 d\Omega^2). \tag{12}$$

Similar to the McVittie solution, the following matter distribution in the universe is assumed: A spherical symmetric distribution of matter around the origin where there is a wormhole, no flow of the matter as a whole either towards or away from the origin, and isotropic pressure of matter in the universe. In addition to these assumptions, we add one more assumption that the local and the global matters are separate. Since the wormhole is a localized object, the matter of wormhole is separated from the cosmic matter, which is distributed over the universe. When we adopt the ansatz, the cosmological matter term is time-dependent and isotropic, while the wormhole matter term depends only on space and not necessarily isotropic as,

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