Physics Letters B 779 (2018) 136–142

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Lorentz and diffeomorphism violations in linearized gravity

V. Alan Kostelecký [a,*], Matthew Mewes [b]

[a] Physics Department, Indiana University, Bloomington, IN 47405, USA
[b] Physics Department, California Polytechnic State University, San Luis Obispo, CA 93407, USA

ARTICLE INFO

Article history:
Received 2 January 2018
Received in revised form 30 January 2018
Accepted 30 January 2018
Available online 7 February 2018
Editor: A. Ringwald

ABSTRACT

Lorentz and diffeomorphism violations are studied in linearized gravity using effective field theory. A classification of all gauge-invariant and gauge-violating terms is given. The exact covariant dispersion relation for gravitational modes involving operators of arbitrary mass dimension is constructed, and various special limits are discussed.

The foundational symmetries of General Relativity (GR) include diffeomorphisms and local Lorentz transformations. The former act on the spacetime manifold, while the latter act in the tangent space. These two types of transformations are partially linked through the vierbein, which provides a tool for moving objects between the manifold and the tangent space. The proposal that Lorentz invariance might be broken in an underlying theory of gravity and quantum physics such as strings [1,2] naturally raises various questions about the relationship between diffeomorphism violation and Lorentz violation and about the associated phenomenological signals. These questions can be studied independently of specific models using gravitational effective field theory [3]. Here, following a brief summary of the current status and results, we develop a model-independent framework for studying these issues in linearized gravity. This limit provides a comparatively simple arena for exploration, and it is crucial for experimental analyses of gravitational waves and of gravitation in the Newton and post-Newton limits.

A generic treatment of Lorentz violation in Minkowski spacetime in the absence of gravity is comparatively straightforward using effective field theory [4]. In this context, the role of diffeomorphisms and local Lorentz transformations is played by translations and Lorentz transformations that act globally and combine to form the Poincaré group. The two symmetries can be broken independently, and a physical breaking of either one can be represented in terms of nonzero background fields in an effective field theory. The breaking of either can be spontaneous or explicit. Spontaneous breaking occurs when the background is dynamical, which means that it must satisfy the equations of motion and that it comes with fluctuations in the form of Nambu–Goldstone modes [5] and possibly also massive modes. In most applications of spontaneous breaking, the background satisfies the equations of motion in vacuum and can therefore be viewed as the vacuum expectation value. In contrast, explicit breaking is a consequence of a prescribed background, which is typically off shell and has no associated fluctuations. Much of the phenomenological literature investigating Lorentz violation in Minkowski spacetime assumes for simplicity that global spacetime translations are preserved in an approximately local inertial frame, canonically taken to be the Sun-centered frame [6]. This guarantees conservation of energy and momentum, so phenomenological signals are restricted to violations of the conservation laws for generalized angular momenta. A large body of experimental studies constrains this type of Lorentz violation [7].

In the presence of gravity, the situation becomes more involved. One complication arises because diffeomorphisms and local Lorentz transformations act on objects in different spaces that can be linked via the vierbein, which can relate the corresponding violations. In the case of spontaneous breaking, for example, the vacuum expectation values are on shell and a nonzero background on the spacetime manifold implies one in the tangent space and vice versa. As a result, diffeomorphism violation occurs if and only if local Lorentz violation does [8]. More intuitively, local Lorentz violation can be understood as a background direction dependence in a local freely falling frame [3]. Transporting this to the spacetime manifold via the vierbein then guarantees the existence of a direction dependence on the spacetime manifold and hence diffeomorphism violation.

Another complication for gravity concerns conservation laws and arises from the difference between spontaneous and explicit breaking. In general, a theory invariant under local transforma-

* Corresponding author.
E-mail address: kostelec@indiana.edu (V.A. Kostelecký).

https://doi.org/10.1016/j.physletb.2018.01.082
0370-2693/

tions comes with covariantly conserved currents [9]. In spontaneous breaking, the full theory remains invariant under the transformations and the symmetry is only hidden [10]. The currents remain conserved even though the background is unchanged by the transformations because the background fluctuations transform in a nonstandard way to compensate. This contrasts with explicit breaking, when the current conservation laws fail to hold.

In GR, local Lorentz invariance implies symmetry of the energy–momentum tensor while diffeomorphism invariance implies its covariant conservation [11]. In theories with spontaneous diffeomorphism and local Lorentz violation, these current-conservation laws are unaffected: an energy–momentum tensor for the full theory remains covariantly conserved and it is always possible to make it symmetric [3]. However, if explicit breaking occurs, then there is no guarantee that the energy–momentum tensor is explicitly conserved or symmetric, and as a result a theory with explicit breaking can be inconsistent or require reformulation within Finsler geometry [3,12]. For sufficiently involved models, this situation can be rescued by the additional modes that appear in theories with explicit diffeomorphism and local Lorentz violation [13]. These additional modes arise because in explicit breaking it becomes impossible to remove all four diffeomorphism degrees of freedom and six local Lorentz degrees of freedom from the vierbein. In some models, these additional modes can be constrained to restore the covariant conservation and symmetry of the energy–momentum tensor. The additional modes are the counterparts in explicit breaking of the Nambu–Goldstone modes appearing in spontaneous breaking. Indeed, they can be understood as Nambu–Goldstone excitations of Stueckelberg fields [14,15].

The above results have several implications for the phenomenology of diffeomorphism and local Lorentz violations in gravity. If the breaking is explicit, the challenge lies in establishing the consistency of theory and, if achieved, then in determining the effects of the additional modes on observational signals. In contrast, if the breaking is spontaneous, the Nambu–Goldstone and massive fluctuations can play the role of new forces affecting the phenomenology and so must be taken into account in analyzing experimental signals. Model-independent techniques for this have been developed both in the pure-gravity and in the matter-gravity sectors [16–26] and applied to obtain model-independent constraints on diffeomorphism and local Lorentz violation in gravity from a variety of experimental tests [7,27–56].

An alternative model-independent approach to studying both spontaneous and explicit diffeomorphism and local Lorentz violation uses linearized effective field theory for gravity, formulated to incorporate gauge and Lorentz violation [46]. In this context, gauge transformations are linearized diffeomorphisms of the metric fluctuation. This technique yields an explicit construction and classification of the general quadratic Lagrange density in effective field theory with gauge invariance at linearized level. It also permits construction of the general covariant dispersion relation and investigation of the properties of the corresponding gravitational modes. These results have been applied to obtain model-independent constraints on linearized coefficients for Lorentz violation using gravitational waves [46,51] and tests of gravity at short range [52,53]. In the present work, we extend this approach to explicit gauge breaking. We construct and classify all terms for the quadratic Lagrange density in gravitational effective field theory with explicit gauge violation, and we derive the corresponding covariant dispersion relation required for experimental applications. Throughout the work, we adopt the conventions of Ref. [3]: the metric signature is +2, the Levi-Civita tensor satisfies $\epsilon_{0123} = +1$, and parentheses or brackets about indices indicate symmetrization or antisymmetrization without numerical factors.

To perform the linearization, we expand the dynamical metric $g_{\mu\nu}$ in a flat-spacetime background with Minkowski metric, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. A generic term of mass dimension $d \geq 2$ in the Lagrange density for the linearized gravitational effective field theory can then be written as

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \tfrac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}, \tag{1}$$

where $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ is the product of a coefficient $\mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\ldots\varepsilon_{d-2}}$ with $d-2$ derivatives $\partial_{\varepsilon_1}\partial_{\varepsilon_2}\ldots\partial_{\varepsilon_{d-2}}$. The coefficients $\mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\ldots\varepsilon_{d-2}}$ have mass dimension $4-d$ and are assumed constant and small. The complete traces of these coefficients control Lorentz-invariant terms in $\mathcal{L}_{\mathcal{K}^{(d)}}$, while the other components govern Lorentz violation. To contribute nontrivially to the equations of motion, the operator $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ must satisfy the requirement $\hat{\mathcal{K}}^{(d)(\mu\nu)(\rho\sigma)} \neq \pm\hat{\mathcal{K}}^{(d)(\rho\sigma)(\mu\nu)}$, where the upper sign holds for odd d and the lower one for even d.

The action is invariant under the usual gauge transformations $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$ when the condition $\hat{\mathcal{K}}^{(d)(\mu\nu)(\rho\sigma)}\partial_\nu = \pm\hat{\mathcal{K}}^{(d)(\rho\sigma)(\mu\nu)}\partial_\nu$ holds. Assuming this condition, the operators $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ can be constructed explicitly, using standard methods in group theory [57]. They are found to span three representation classes [46]. For the present work, we have extended this construction by relaxing the requirement of gauge invariance. Decomposing the operator $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ into irreducible pieces then yields another 11 representation classes. This shows that a total of only 14 independent classes of operators can appear in any linearized gravitational effective field theory, whether or not the Lorentz and gauge invariances hold. These 14 classes therefore characterize all phenomenological effects in linearized gravity, including effects on the propagation of gravitational waves and in the Newton and post-Newton limits.

To simplify the notation in what follows, we denote indices contracted into a derivative as a circle index $\circ$, with n-fold contractions denoted as $\circ^n$. With this convention, the generic operator $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ can be written as $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma\circ^{d-2}}$. Also, we denote the 14 representation classes as indicated in the first column of Table 1. To obtain the term in the Lagrange density (1) associated to a given class, it suffices to replace $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ with the operator listed. The second column displays the index symmetries of each class using Young tableaux. The Table also lists some properties of each class. The third column indicates whether the operator is fully gauge invariant, and the fourth column displays the handedness under CPT of the associated term in the Lagrange density. Each class can occur only for even or for odd d and for d above a minimal value, as shown in the next column. The final column lists the total number of independent components appearing in the coefficient $\mathcal{K}^{(d)\mu\nu\rho\sigma\varepsilon_1\varepsilon_2\ldots\varepsilon_{d-2}}$ for fixed d.

The quadratic approximation $\mathcal{L}_0$ to the Lagrange density for the Einstein–Hilbert action can conveniently be written in the form

$$\mathcal{L}_0 = \tfrac{1}{4}\epsilon^{\mu\rho\alpha\kappa}\epsilon^{\nu\sigma\beta\lambda}\eta_{\kappa\lambda}h_{\mu\nu}\partial_\alpha\partial_\beta h_{\rho\sigma}. \tag{2}$$

This gauge- and Lorentz-invariant term is constructed from a piece of the coefficient $s^{(4)\mu\rho\alpha\nu\sigma\beta}$ in the first line of Table 1. The complete Lagrange density incorporating all the operators in Table 1 can then be expressed as

$$\mathcal{L} = \mathcal{L}_0 + \tfrac{1}{4}h_{\mu\nu}\sum_{\mathcal{K},d}\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}h_{\rho\sigma}, \tag{3}$$

where the sum is over all the representation classes $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma}$ shown in Table 1 and also over all allowed dimensions d for each class. Larger values of d introduce higher powers of momenta and