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Does the detection of primordial gravitational waves exclude low energy inflation?

Tomohiro Fujita [a,b], Ryo Namba [c], Yuichiro Tada [d,e,f,g]

[a] Stanford Institute for Theoretical Physics, Stanford University, Stanford, CA 94306, USA
[b] Department of Physics, Kyoto University, Kyoto, 606-8502, Japan
[c] Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada
[d] Institut d'Astrophysique de Paris, UMR-7095 du CNRS, Université Pierre et Marie Curie, 98 bis bd Arago, 75014 Paris, France
[e] Sorbonne Universités, Institut Lagrange de Paris, 98 bis bd Arago, 75014 Paris, France
[f] Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan
[g] Institute for Cosmic Ray Research, The University of Tokyo, Kashiwa, Chiba 277-8582, Japan

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ABSTRACT

We show that a detectable tensor-to-scalar ratio ($r \geq 10^{-3}$) on the CMB scale can be generated even during extremely low energy inflation which saturates the BBN bound $\rho_{\rm inf} \approx (30\ {\rm MeV})^4$. The source of the gravitational waves is not quantum fluctuations of graviton but those of $SU(2)$ gauge fields, energetically supported by coupled axion fields. The curvature perturbation, the backreaction effect and the validity of perturbative treatment are carefully checked. Our result indicates that measuring r alone does not immediately fix the inflationary energy scale.

1. Introduction

The inflationary paradigm has been successful over the past few decades to serve as a mechanism to produce the observed inhomogeneities in the universe such as the cosmic microwave background (CMB) anisotropies and large-scale structure (LSS), while resolving the conceptual difficulties in the hot big bang scenario. An important prediction in the framework is generation of the B-mode polarization in the CMB [1], whose signal is conventionally quantified by the tensor-to-scalar ratio $r \equiv \mathcal{P}_h/\mathcal{P}_\zeta|_{k=k_{\rm CMB}}$. The current bound is $r < 0.07$ at $k_{\rm CMB} = 0.05\,{\rm Mpc}^{-1}$ with 95% confidence [2], and a number of proposed missions are expected to improve the bound to $\mathcal{O}(10^{-3})$ (see e.g. [3]). The conventional relationship between the tensor-to-scalar ratio and the Hubble parameter during inflation is

$$r_{\rm vac} = \mathcal{P}_\zeta^{-1}\frac{2H_{\rm inf}^2}{\pi^2 M_{\rm Pl}^2} \approx 10^{-3}\left(\frac{H_{\rm inf}}{8\times 10^{12}{\rm GeV}}\right)^2 , \tag{1}$$

where $H_{\rm inf}$ is the Hubble parameter during inflation and $\mathcal{P}_\zeta \approx 2.2\times 10^{-9}$ has been used [4]. An immediate implication of (1) is that detection of r would fix the inflationary scale at such high energy levels as beyond our current experimental reach.

Considering the ongoing and upcoming experimental efforts for B-mode detection, it is right time to test the validity of the conventional prediction (1). In general, the value of r at cosmological scales can be estimated as the spectrum of the energy fraction of gravitational wave (GW) at the horizon crossing divided by $\mathcal{P}_\zeta$

$$r \simeq \mathcal{P}_\zeta^{-1}\frac{1}{\rho_{\rm inf}}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k}\bigg|_{k=aH_{\rm inf}}, \tag{2}$$

where $\rho_{\rm inf} \equiv 3M_{\rm Pl}^2 H_{\rm inf}^2$ and ${\rm d}\rho_{\rm GW}/{\rm d}\ln k \simeq H^2 M_{\rm Pl}^2 \mathcal{P}_h$ at the horizon crossing. The energy density of GW from the vacuum fluctuations produced during the quasi de Sitter expansion must be characterized by the Hubble scale ${\rm d}\rho_{\rm GW}^{\rm vac}/{\rm d}\ln k \simeq H_{\rm inf}^4$, leading to the conventional relation $r_{\rm vac} \propto H_{\rm inf}^2$.

On the other hand, if GW is induced by another energy source, the conventional relation (1) may be altered. Provided that an energy source ρ_s generates GWs with efficiency γ, one generally expects

$$r \simeq \mathcal{P}_\zeta^{-1}\frac{\gamma}{\rho_{\rm inf}}\frac{{\rm d}\rho_s}{{\rm d}\ln k}\bigg|_{k=aH_{\rm inf}}, \tag{3}$$

which can be significant even if $\rho_s \ll \rho_{\rm inf}$ and $\gamma \ll 1$ thanks to the smallness of $\mathcal{P}_\zeta$. Conventionally, however, an efficient energy transfer from a source to GW has been assumed to be rather difficult. The reasoning is rooted in the *decomposition theorem* in

E-mail address: t.fujita@tap.scphys.kyoto-u.ac.jp (T. Fujita).

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cosmology, which states that perturbations around a homogeneous and isotropic background can be decomposed into scalar, vector and tensor sectors that are mutually decoupled at the linearized order. Since GW is the only tensor degree of freedom in the Einstein gravity, we have no choice but use the source term from scalar δS or vector perturbation δV_i which is schematically written as

$$\Box h_{ij}(t,\boldsymbol{x}) = O^{(S)}_{ij}(t,\boldsymbol{\partial})\,\delta S(t,\boldsymbol{x}) + O^{(V)}_{ijk}(t,\boldsymbol{\partial})\,\delta V_k(t,\boldsymbol{x})\,, \tag{4}$$

where $O^{(S)}_{ij}$ and $O^{(V)}_{ijk}$ are operators traceless and transverse in the indices ij that depend on time and spatial derivatives. However, the decomposition theorem bans the existence of such operators at the linear order. Although the second order effects (e.g. $\partial_i\delta S\partial_j\delta S,\ \delta V_i\delta V_j$) are allowed to generate GW, the efficiency of the energy transfer is suppressed, because the coefficients of the source term effectively becomes the order of perturbation, $O^{(S)}_{ij}, O^{(V)}_{ijk} = \mathcal{O}(\delta S, \delta V_j)$ [5].

There is a loophole in this argument. If $O^{(V)}_{ijk}$ in (4) consists of the background vector field $\bar{V}_i(t)$, GW can be sourced at linear order by $\bar{V}_i\delta V_j$. It is known that $SU(2)$ gauge fields can achieve this without disrupting background isotropy by taking a particular configuration.[1] Moreover this isotropic configuration is realized as an attractor solution, if $SU(2)$ gauge fields are coupled to a rolling pseudo-scalar field [6]. Therefore $SU(2)$ gauge fields can source the GW through the terms $\bar{V}_i\delta V_j$ without violating the isotropy of the universe at the linear order, thus with a high efficiency of the energy transfer.

As we shall see later, the energy source ρ_s to generate GW is the (linear) perturbation of a $SU(2)$ gauge field. It is produced as quantum fluctuations and thus acquires the amplitude $\mathcal{O}(H_{\rm inf})$ around the horizon crossing. In addition, however, it experiences a transient instability around horizon crossing and is amplified by an exponential factor. As a result, the energy fraction of the source and the efficiency factor of energy transfer in (3) are given by

$$\frac{1}{\rho_{\rm inf}}\frac{{\rm d}\rho_s}{{\rm d}\ln k} \sim \frac{H^2_{\rm inf}}{M^2_{\rm Pl}}e^{4m_Q}\,, \quad \gamma \sim \frac{\rho_A}{\rho_{\rm inf}} \equiv \Omega_A\,, \tag{5}$$

where s now denotes the perturbation of $SU(2)$ gauge field, ρ_A is its background energy density, and m_Q is the $SU(2)$ mass parameter in the units of $H_{\rm inf}$. For values of m_Q with $H_{\rm inf}\sqrt{\Omega_A}\,e^{2m_Q} \gtrsim \mathcal{O}(10^{12})\,$GeV, one can realize a detectable r even in the case of low-energy inflation.

In this letter, we seek the lowest possible inflation energy scale with which $SU(2)$ gauge fields produce primordial GWs detectable by the upcoming observations (i.e. $r \geq 10^{-3}$). Although this GW generation mechanism has been studied in previous works [6–8], it was not revealed to what extent the tensor-to-scalar ratio can be enhanced. To this end, for the first time, we numerically solve the background and perturbations taking into account the backreaction. We also quantify the effect on the scalar tilt n_s, and verify the perturbative treatment by calculating the 1loop correction of the $SU(2)$ perturbation.

2. Spectator axion-$SU(2)$ model

In our consideration of GW production, we leave the gravity sector as the standard Einstein–Hilbert and the inflation model unspecified, which is also responsible for generating the observed curvature perturbation. We then consider the axion-$SU(2)$ sector with the action [8] (see also [9]):

$$\mathcal{L}_{\chi A} = -\frac{1}{2}(\partial_\mu\chi)^2 - V(\chi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda}{4f}\chi F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\,, \tag{6}$$

where χ is a pseudo-scalar field (axion) with a cosine-type potential $V(\chi) = \mu^4[1+\cos(\chi/f)]$ with dimensionful parameters μ and f, $F^a_{\mu\nu} \equiv 2\partial_{[\mu}A^a_{\nu]} - g\epsilon^{abc}A^b_\mu A^c_\nu$ and $\tilde{F}^{a\mu\nu}$ are the field strength of $SU(2)$ gauge field and its dual, respectively, and λ is a dimensionless coupling constant.

At the background level, it is shown that the isotropic configuration of the $SU(2)$ gauge fields, $A^a_0 = 0$ and $A^a_i = \delta^a_i a(t)Q(t)$, is an attractor solution while the vev of $\chi(t)$ slowly rolls down its potential [6,8]. At the perturbation level, δA^a_μ contains two scalar δQ, M, two vector M_i and two tensor t_{ij} polarizations as dynamical degrees of freedom [6,8]. Interestingly, t_{ij} is coupled to the metric tensor modes h_{ij} already at the linear order, and only one circular polarization mode of t_{ij} is substantially amplified due to a transient instability around the horizon crossing. It then efficiently sources only one polarization of GW h_{ij} at the linear order, if $m_Q \equiv gQ/H > \sqrt{2}$ [7]. Therefore we focus on t_{ij} among the perturbations of A^a_μ.

The Einstein equation at the background yields

$$3M^2_{\rm Pl}H^2 = \rho_\phi + \rho_\chi + \rho_A + \rho_t\,, \tag{7}$$

$$-\dot{H}/H^2 = \epsilon_\phi + \epsilon_\chi + \epsilon_A + \epsilon_t\,, \tag{8}$$

where $\rho_\chi = \dot{\chi}^2/2 + V(\chi)$, $\rho_A = 3\epsilon_A M^2_{\rm Pl}H^2/2$, $\epsilon_A = \epsilon_E + \epsilon_B$, $\epsilon_E \equiv (\dot{Q}+HQ)^2/M^2_{\rm Pl}H^2$, $\epsilon_B \equiv g^2Q^4/M^2_{\rm Pl}H^2$, $\epsilon_\chi = \dot{\chi}^2/2M^2_{\rm Pl}H^2$, and dot denotes the cosmic time derivative. The inflaton part ρ_ϕ and $\epsilon_\phi \equiv -\dot{\rho}_\phi/6M^2_{\rm Pl}H^3$ depend on the inflation model, and ρ_t and $\epsilon_t \equiv -\dot{\rho}_t/6M^2_{\rm Pl}H^2$ denote the contributions from the perturbation t_{ij} on the background dynamics, which will be discussed later. The equations of motion for $\chi(t)$ and $Q(t)$ are

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\mu^4}{f}\sin\left(\frac{\chi}{f}\right) + \frac{3g\lambda}{f}Q^2\left(\dot{Q}+HQ\right) + \mathcal{T}^\chi_{BR} = 0, \tag{9}$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H}+2H^2\right)Q + 2g^2Q^3 - \frac{g\lambda}{f}Q^2\dot{\chi} + \mathcal{T}^Q_{BR} = 0, \tag{10}$$

where we include the backreaction terms, $\mathcal{T}^Q_{BR}$ and $\mathcal{T}^\chi_{BR}$, from t_{ij}. Without the backreaction, one can show that the effective potential of Q uplifted by the coupling to χ acquires a non-zero minimum at $Q_{\rm min} \simeq \left(\mu^4\sin(\chi/f)/3g\lambda H\right)^{1/3}$, if χ slowly rolls and the coupling is sufficiently strong [6,8].

The tensor perturbations consist of t_{ij} and h_{ij}, and each of them can be decomposed into the circular polarization modes $t_{R/L}$ and $h_{R/L}$, respectively. At the linearized order, one finds their equations of motion coupled together among the same polarizations, written in the Fourier space as [8],

$$\partial^2_x t_{R,L} + \left[1 + \frac{2m_Q\xi}{x^2} \mp 2\frac{m_Q+\xi}{x}\right]t_{R,L} \approx 0 \tag{11}$$

$$\partial^2_x\psi_{R,L} + \left(1 - \frac{2}{x^2}\right)\psi_{R,L} \approx \mathcal{S}^\psi_{R,L}\,, \tag{12}$$

where $x \equiv k/aH$ and $\psi_{R,L}(t,k)$ are the mode functions of the canonical gravitational wave, $\psi_{ij} \equiv aM_{\rm Pl}h_{ij}/2$. While $t_{R/L}$ are sourced by $\psi_{R/L}$ in principle, the former is always parametrically larger than the latter for our concern, and thus ignoring the right-hand side of (11) is a justified approximation. We have also neglected slow-roll suppressed and subdominant terms in (11) and

[1] This does not restrict possible models to those with $SU(2)$ only, as long as the symmetry in the models allow this configuration.