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The nuclear contacts and short range correlations in nuclei

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ABSTRACT

Atomic nuclei are complex strongly interacting systems and their exact theoretical description is a long-standing challenge. An approximate description of nuclei can be achieved by separating its short and long range structure. This separation of scales stands at the heart of the nuclear shell model and effective field theories that describe the long-range structure of the nucleus using a mean-field approximation. We present here an effective description of the complementary short-range structure using contact terms and stylized two-body asymptotic wave functions. The possibility to extract the nuclear contacts from experimental data is presented. Regions in the two-body momentum distribution dominated by high-momentum, close-proximity, nucleon pairs are identified and compared to experimental data. The amount of short-range correlated (SRC) nucleon pairs is determined and compared to measurements. Non-combinatorial isospin symmetry for SRC pairs is identified. The obtained one-body momentum distributions indicate dominance of SRC pairs above the nuclear Fermi-momentum.

The atomic nucleus is one of the most complex systems in nature. One of the main challenges in describing nuclei is understanding the short interparticle part of the nuclear wave function. The challenge stems from the complicated nucleon–nucleon interaction and the large density of the nucleus. The latter causes all the relevant scales of the system (nucleon size, average distance, and interaction range) to be comparable, making effective theoretical descriptions very demanding. On the other hand, detailed understanding of these short-range correlations (SRCs) is important for neutron-star structure and the nuclear symmetry energy [1–4], the bound nucleon and free neutron structure functions [5–10], neutrino-nucleus interactions and neutrino oscillation experiments [11–15], and more.

Current mean-field nuclear theories describe well various static properties of nuclei, but fail to describe the dynamic effects of SRCs. Ab-initio many-body calculations [16–21] are still limited to light nuclei and/or to soft interactions that regulate the short-range/high-momentum parts of the nuclear wave function. Therefore, effective theories are still needed to describe medium and heavy nuclei and to identify the main physical process at short distances [22–25].

In the last decade there was a significant progress in describing SRCs in dilute Fermi systems. It was shown that if the interaction range r_0 is much shorter than the average interparticle distance d, and the scattering length a_s, a *contact* theory can be used to describe the system [26–29]. A series of relations between different observables and the probability of finding a particle pair in a close proximity emerge. The contact theory was studied in great detail theoretically, and validated experimentally, for ultra-cold Fermi gases [26–36].

For nuclei, several experimental observations resemble those of cold atomic systems [37,38]. However, in nuclei, the short-range interaction is about 0.5–1.5 fm, the average distance between nucleons is about 2.5 fm, and the scattering length is about −20 fm and 5 fm for the spin-singlet and spin-triplet channels, respectively. Therefore, the possibility to generalize the contact theory to nuclear systems is not obvious. Nevertheless, a generalized nuclear theory was recently presented which addresses the factorization of the nuclear many-body wave function at short distances [39]. Few of its predictions were verified, yet more convincing theoretical and experimental results must be provided to prove that indeed it is adequate for describing nuclear SRCs.

Many features of nuclear SRCs are well known and should be properly explained by any candidate theory. Recent scattering experiments indicate that SRC pairs account for 20%–25% of the nu-

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cleons in the nucleus and practically all nucleons with momentum above the Fermi momentum (k_F) [40–48]. They are predominantly in the form of neutron–proton (np) SRC pairs with large relative momentum ($k > k_F$), and small center-of-mass (c.m.) momentum ($K < k_F$). Here, $k_F \sim 255$ MeV/$c = 1.3$ fm^{-1} is the typical Fermi momentum of medium and heavy nuclei. These, and results of theoretical studies, indicate that the high-momentum ($k > k_F$) tail of the nuclear momentum distribution is dominated by SRC and described using a factorized wave function for the c.m. and relative momentum distributions of the pairs which results in similar two-body densities for different nuclei [22–25,49–54]. For recent reviews see [8,55]. Between these well-established properties and the generalized contact formalism there is a seemingly unsolved tension, as the latter's predictions involving two-body momentum distributions, are only satisfied for very high momentum, $k > 3k_F \approx 4$ fm^{-1}, and not for lower momentum $k_F < k < 3k_F$.

In this work, we will show that the generalized nuclear contact formalism can indeed describe SRCs in nuclei also in this lower momentum range. A direct agreement with both recent experimental data and with variational Monte Carlo (VMC) calculations will be presented. We will also discuss the nontrivial manner in which information on SRC is encapsulated in the nuclear two-body momentum distributions. The values of the nuclear contacts for various nuclei will be extracted using the VMC two-body distributions in coordinate and momentum space, separately, and also using experimental data. We find all three approaches to yield consistent values. Last, the VMC one-body momentum distributions are compared to the contact-formalism predictions, confirming the experimental observation that they are dominated by SRCs for momentum larger than k_F.

Generalized contact theory for nuclei – The original contact theory was formulated for systems with significant scale separation. Consequently, the Bethe–Peierls boundary condition can be used, leading at short interparticle distance to a factorized asymptotic wave-function of the form [29]:

$$\Psi \xrightarrow{r_{ij}\to 0} \varphi(\boldsymbol{r}_{ij}) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}\}_{k\neq ij}). \tag{1}$$

Here $\varphi(\boldsymbol{r}_{ij})$ is an asymptotic two-body wave function, and A_{ij} is a function of the residual $A-2$ particle system. The scale separation allows replacing the short-range interaction with a boundary condition, and to ignore all partial waves but s-wave, leading to $\varphi(\boldsymbol{r}_{ij}) = (1/r_{ij} - 1/a_s)$. In momentum space, this factorized wave function leads to a high momentum tail, valid for $|a_s|^{-1}, d^{-1} \ll k \ll r_0^{-1}$, that is given by: $n(k) \to C/k^4$, where $C = 16\pi^2 \sum_{ij} \langle A_{ij}|A_{ij}\rangle$ is known as the *contact*.

To generalize this formalism to nuclear systems we need to consider two main points: (1) different partial waves might be significant, and therefore a sum over all possible nucleon–nucleon channels α must be introduced, and (2) as full scale separation does not exist, the asymptotic two-body channel wave-functions φ_α are taken from the solution of the nuclear zero-energy two-body problem. Therefore, the factorized asymptotic wave-function takes the form

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}\}_{k\neq ij}) \ , \tag{2}$$

similar to the independent-pair approximation [56], where the index ij corresponds to pn, pp, and nn pairs [57].

In this work we will consider only the main channels contributing to SRCs, namely, the pn deuteron channel ($\ell = 0, 2$ and $s = 1$ coupled to $j = 1$) and the singlet pp, pn, and nn s-wave channel ($\ell = s = j = 0$). Using Eq. (2), asymptotic expressions for the one- and two-body momentum densities can be derived [39]:

$$\begin{aligned} n_p(\boldsymbol{k}) =& 2C_{pp}^{s=0}|\tilde{\varphi}_{pp}^{s=0}(\boldsymbol{k})|^2 + C_{pn}^{s=0}|\tilde{\varphi}_{pn}^{s=0}(\boldsymbol{k})|^2 \\ &+ C_{pn}^{s=1}|\tilde{\varphi}_{pn}^{s=1}(\boldsymbol{k})|^2 \end{aligned} \tag{3}$$

$$\begin{aligned} F_{pp}(\boldsymbol{k}) =& C_{pp}^{s=0}|\tilde{\varphi}_{pp}^{s=0}(\boldsymbol{k})|^2 \\ F_{pn}(\boldsymbol{k}) =& C_{pn}^{s=0}|\tilde{\varphi}_{pn}^{s=0}(\boldsymbol{k})|^2 + C_{pn}^{s=1}|\tilde{\varphi}_{pn}^{s=1}(\boldsymbol{k})|^2 \end{aligned} \tag{4}$$

and the same when replacing n with p. Here, C_{ij}^α are the nuclear contacts that determine the number of pairs in a given two-body channel, $n_N(\boldsymbol{k})$ is the one-body momentum distribution, and $F_{NN}(\boldsymbol{k})$ is the relative two-body momentum distribution. $F_{NN}(\boldsymbol{k}) = \int d\boldsymbol{K} F_{NN}(\boldsymbol{k}, \boldsymbol{K})$, where $F_{NN}(\boldsymbol{k}, \boldsymbol{K})$ is the probability of finding a pair of nucleons with relative momentum $\boldsymbol{k}$, and center-of-mass (c.m.) momentum $\boldsymbol{K}$. Similarly, $\rho_{NN}(\boldsymbol{r})$ describes the probability to find a pair of nucleons with relative distance $\boldsymbol{r}$. The subscripts N, and NN, stand for the type of nucleon/nucleon-pairs considered. Clearly, $n_{p(n)}(\boldsymbol{k}) = 2F_{pp(nn)}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$ [39]. Equivalent two-body coordinate space densities for $\rho_{NN}(\boldsymbol{r})$ are given by replacing $\tilde{\varphi}(\boldsymbol{k})$ with $\varphi(\boldsymbol{r})$ in Eq. (4), while keeping the same nuclear contacts. We note that in deriving Eq. (3) the center-of-mass momentum of the pairs was assumed to be much smaller than k.

We choose to normalize $\tilde{\varphi}(\boldsymbol{k})$ such that $\int_{k_F}^\infty |\tilde{\varphi}(\boldsymbol{k})|^2 d\boldsymbol{k} = 1$. Using this normalization, and Eq. (3), the fraction of the one-body momentum density above k_F is given by:

$$\frac{\int_{k_F}^\infty n(\boldsymbol{k})d\boldsymbol{k}}{\int_0^\infty n(\boldsymbol{k})d\boldsymbol{k}} = \frac{C_{nn}^{s=0} + C_{pp}^{s=0} + C_{pn}^{s=0} + C_{pn}^{s=1}}{A/2}, \tag{5}$$

where $n(\boldsymbol{k}) = n_n(\boldsymbol{k}) + n_p(\boldsymbol{k})$, A is the number of nucleons in the nucleus and $C_{NN}^s/(A/2)$ gives the fraction of the one-body momentum density above the Fermi momentum due to each type of SRC pair.

Ab-initio nuclear two-body densities – Recent progress in quantum Monte-Carlo techniques allows performing ab-initio many-body calculations of nuclear structure for nuclei as heavy as ^{12}C [16,17]. Furthermore, cluster variational Monte-Carlo (CVMC) provides a way to obtain nuclear structure calculations for ^{16}O and ^{40}Ca [21]. These calculations are done using the AV18 and UX potentials, and result in one- and two-body nucleon densities in coordinate and momentum space.

The detailed study of the relation between two-body densities and two-nucleon knockout measurements is only now starting [39, 49,50].

When examining the two-body densities at high relative momentum, certain care should be taken to separate SRC pairs from non-correlated pairs with high relative momentum. Two nucleons that form an SRC pair are close to each other, each have high momentum, their relative momentum is high, and their c.m. momentum is low. However, not all nucleon pairs with high relative momentum are necessarily SRC pairs. For example, a particle with momentum $k_1 \approx 4k_F$, and any uncorrelated "mean-field" particle at rest $k_2 \approx 0$, will yield a pair with high relative momentum $k \approx 2k_F$, and c.m. momentum $K \approx 2k_F$. In such cases, the high c.m. momentum is a signature for uncorrelated pairs. As we examine pairs with larger and larger relative momentum, this scenario becomes less and less probable as the probability of finding a nucleon with high momentum falls fast with the momentum, i.e. its easier to find two nucleon with momentum $\approx 2k_F$ then one nucleon with momentum $\approx 4k_F$.

There are two ways to access regions in the two-body momentum distribution dominated by SRC pairs, with minimal mean-field nucleon contamination. One is to integrate over the pairs c.m. momentum but request a very large relative momentum, which ensures that the pair is truly an SRC pair. This explains why Ref. [39] observed scaling between the one and two-body densities only for momentum much larger than k_F. The alternative approach is to